Chapter 2

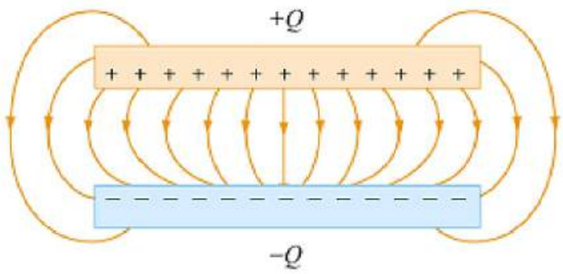
Electrostatic Potential and Capacitance

Capacitance & Electrostatic Potential

What is a Capacitor?

Capacitors are also known as Electric-condensers. A capacitor is a two-terminal electric component. It has the ability or capacity to store energy in the form of electric charge.

- The capacitor is an arrangement of two conductors generally carrying charges of equal magnitudes and opposite sign and separated by an insulating medium.
- A capacitor is a device that stores electric charge.
- Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges.



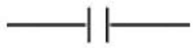
What are Capacitors Used For?

- Capacitors have many important applications in electronics. Some examples include storing electric potential energy, delaying voltage changes when coupled with resistors, filtering out unwanted frequency signals, forming resonant circuits and making frequency-dependent and independent voltage dividers when combined with resistors.
- When charges are pulled apart, energy is associated with the pulling apart of charges, just like energy is involved in stretching a spring thus, some energy is stored in capacitors.
- In the uncharged state, the charge on either one of the conductors in the capacitor is zero. During the charging process a charge, Q is moved from

one conductor to the other one, giving one conductor a charge, +Q and the other one a charge, -Q. A potential difference ΔV is created with the positively charged conductor at a higher potential than the negatively charged conductor. Note that whether charged or uncharged. the net charge on the capacitor as a whole is zero.

Note:

- The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge Q, we mean that the positively charged conductor has charge +Q and negatively charged conductor has a charge, -Q.
- In a circuit a capacitor is represented by the symbol:



What is Capacitance?

The capacitance of the conductor is defined as the charge required to increase the potential of a conductor by one unit. It is a scalar quantity. Unit of capacitance is farad in SI unis and its dimensional formula is $M^{-1}L^{-2}I^{2}T^{4}$

- Capacitance is nothing but the ability of a capacitor to store the energy in form of an electric charge. In other words, capacitance is the storing ability of a capacitor. It is measured in farads.
- **1 Farad:** 1 Farad is the capacitance of a conductor for which 1-coulomb charge increases potential by 1 volt.

$1\ \text{Farad} = \frac{1\text{Coulomb}}{1\ \text{Volt}}$
$1\ \mu F = 10^{-6}\ F$
$1\ nF = 10^{-9}\ F$
or
$1\ pF = 10^{-12}\ F$

Applications of Capacitors

1. **Capacitors for Energy Storage**

Since the late 18th century, capacitors are used to store electrical energy. Individual capacitors do not hold a great deal of energy, providing only enough power for electronic devices to use during temporary power outages or when they need additional power. There are many applications that use capacitors as energy sources and a few of them are as follows:

- Audio equipment
- Camera Flashes
- Power supplies
- Magnetic coils
- Lasers

Supercapacitors are capacitors that have high capacitances up to 2 kF. These capacitors store large amounts of energy and offer new technological possibilities in areas such as electric cars, regenerative braking in the automotive industry and industrial electrical motors, computer memory backup during power loss, and many others.

2. Capacitors for Power Conditioning
One of the important applications of capacitors is the conditioning of power supplies. Capacitors allow only AC signals to pass when they are charged blocking DC signals. This effect of a capacitor is majorly used in separating or decoupling different parts of electrical circuits to reduce noise, as a result of improving efficiency. Capacitors are also used in utility substations to counteract inductive loading introduced by transmission lines.

3. Capacitors as Sensors
Capacitors are used as sensors to measure a variety of things including humidity, mechanical strain, and fuel levels. Two aspects of capacitor construction are used in the sensing application – the distance between the parallel plates and the material between them. The former is used to detect mechanical changes such as acceleration and pressure and the latter is used in sensing air humidity.

4. Capacitors for Signal Processing
There are advanced applications of capacitors in information technology. Capacitors are used by Dynamic Random Access Memory (DRAM) devices to represent binary information as bits. Capacitors are also used in conjunction with inductors to tune circuits to particular frequencies, an effect exploited by radio receivers, speakers, and analog equalizers.

Factors Affecting Capacitance

1. **Dielectric**
   The effect of dielectric on capacitance is that the greater the permittivity of the dielectric the greater the capacitance, likewise lesser the

permittivity of the dielectric the lesser is the capacitance. Some materials offer less opposition to the field flux for a given amount of field force. Materials with greater permittivity allow more field flux, hence greater charge is collected.

2. **Plate Spacing**

   The effect of spacing on the capacitance is that it is inversely proportional to the distance between the plates. Mathematically it is given as:

$$C \propto \frac{1}{d}$$

3. Area of the Plates

The effect of the area of the plate is that the capacitance is directly proportional to the area. The larger the plate area more is the capacitance value. Mathematically it is given as $C \propto A$

Capacitance of an Isolated Conductor

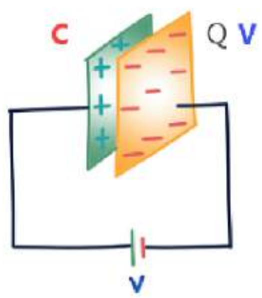
When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to the charge given to it.

q = charge on conductor

V = potential of conductor

⇒ q = CV

Where C is proportionally constant called capacitance of the conductor.



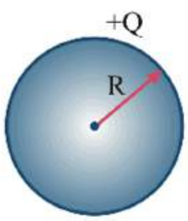
Capacitance of an isolated conductor depends on the following factors

1. **Shape and size of the conductor**
   On increasing the size, capacitance increase.
2. **On surrounding medium**
   With the increase in dielectric constant K, capacitance increases.
3. **Presence of other conductors**
   When a neutral conductor is placed near a charged conductor capacitance of conductors increases.

Capacitance of a conductor does not depend on

1. Charge on the conductor
2. Potential of the conductor
3. The potential energy of the conductor

Ex.1 Find out the capacitance of an isolated spherical conductor of radius R.



Let there is charge Q on the sphere.
Therefore, Potential $V = KQ/R$
Hence by the formula: $Q = CV$
$Q = CKQ/R$
$C = 4\pi\varepsilon_0 R$

- If the medium around the conductor is vacuum or air
  $C_{vacuum} = 4\pi\varepsilon_0 R$
  R = Radius of the spherical conductor. (maybe solid or hollow)
- If the medium around the conductor is a dielectric of constant K from the surface of a sphere to infinity then
  $C_{medium} = 4\pi\varepsilon_0 KR$
- $\frac{C_{medium}}{C_{air / vaccum}} = K =$ dielectric constant

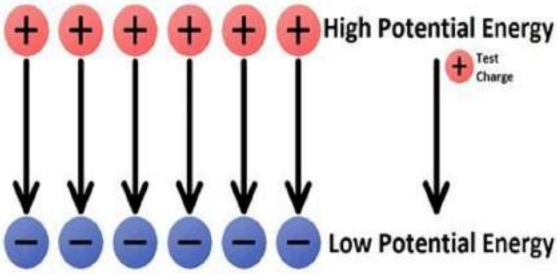
What is Electrostatic Potential?
The electrostatic potential is also known as the electric field potential, electric potential, or potential drop is defined as:

The amount of work that is done in order to move a unit charge from a reference point to a specific point inside the field without producing an acceleration.

- Electric potential is a scalar property of every point in the region of the electric field. At a point in the electric field, the potential is defined as **the interaction energy of a unit positive charge.** If at a point in the electric field a charge q_0 has potential energy U. then the electric potential at that point can be given as: $V = \frac{U}{q_0}$ joule/coulomb
- The potential energy of a charge in an electric field is defined as the work done in bringing the charge from infinity to the given point in the electric field. Similarly, we can define electric potential as work done in bringing a unit positive charge from infinity to the given point against the electric forces.
- Potential at a point can be physically interpreted as the work done within the conservative field in displacing a unit (+ve) charge from infinity to that point. $V = \frac{W}{q_0}$ joule/coulomb

Potential Energy and Potential Difference

- **Electric Potential** is the 'push' of electricity through a circuit. It's easy to confuse electric potential with electric current, so it helps to think of electric current as of the water in our shower and electric potential as the water pressure. Like water pressure, the varying voltage can increase or decrease the flow of electricity.

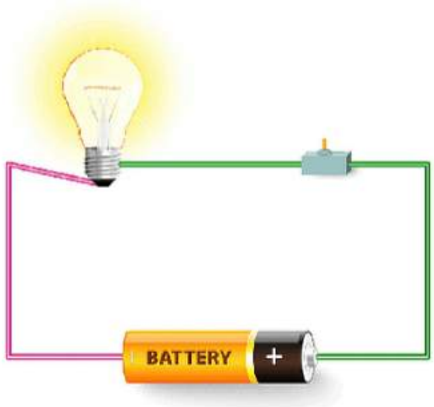


Test charge moves from high potential to low potential

- **Electric Potential** is a scalar quantity denoted by V, equal to the electric potential energy of any charged particle at any location (measured in joules) divided by the charge of that particle (measured in coulombs).

By dividing out the charge on the particle a remainder is obtained that is a property of the electric field itself.

- This value can be calculated in either a static (time-invariant) or a dynamic (varying with time) electric field at a specific time in units of joules per coulomb, or volts (V). The electric potential at infinity is assumed to be zero. The concept of electric potential is useful in understanding electrical phenomena; only differences in potential energy are measurable.
- If an electric field is defined as the force per unit charge, then by analogy an electric potential can be thought of as the potential energy per unit charge. Therefore, the work done in moving a unit charge from one point to another (e.g., within an electric circuit) is equal to the difference in potential energies at each point. In the International System of Units (SI), an electric potential is expressed in units of joules per coulomb (i.e., volts), and differences in potential energy are measured with a voltmeter.

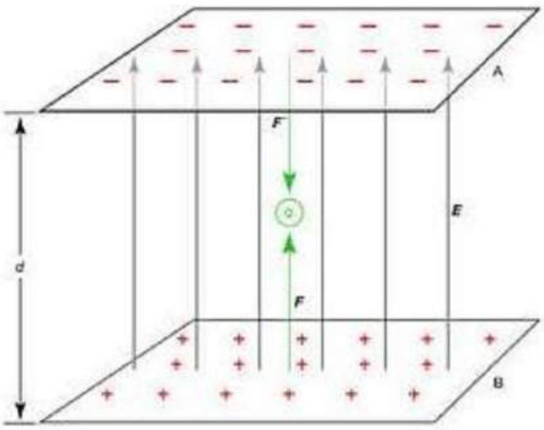


Simple electric circuit

Electric Potential in Circuits

- Charge moving through the wires of the circuit will encounter changes in electric potential as it traverses the circuit. Within the electrochemical cells of the battery, there is an electric field established between the two terminals, directed from the positive terminal towards the negative terminal.

As such, the movement of a positive test charge through the cells from the negative terminal to the positive terminal would require work, thus increasing the potential energy of every Coulomb of charge that moves along this path.



Electric potential movement in circuits

- This corresponds to a movement of the positive charge against the electric field. It is for this reason that the positive terminal is described as the high potential terminal. The charge would lose potential energy as moves through the external circuit from the positive terminal to the negative terminal.
- The negative terminal is described as the low potential terminal. This assignment of high and low potential to the terminals of an electrochemical cell presumes the traditional convention that electric fields are based on the direction of movement of positive test charges.
- Chemical energy is transformed into electric potential energy within the internal circuit (i.e., the battery). Once at the high potential terminal, a positive test charge will then move through the external circuit and do work upon the light bulb or the motor or the heater coils, transforming its electric potential energy into useful forms for which the circuit was designed.
- The positive test charge returns to the negative terminal at low energy and low potential, ready to repeat the cycle (or should we say circuit) all over again.

Electric Potential Difference

- Electric potential energy is defined as the total amount of work done by an external agent in moving a charge from an arbitrarily chosen reference point, which we usually take as infinity, as it is assumed that the electrical potential of a charge at infinity is zero. To that position without any acceleration. For any charge, an electric potential is obtained by dividing the electric potential energy by the quantity of charge.

- In an electrical circuit, the electric potential between two points is defined as the amount of work done by an external agent in moving a unit charge from one point to another.

Mathematically,

E = W/Q

Where,

E = electrical potential difference between two points

W = Work done in moving a charge from one point to another

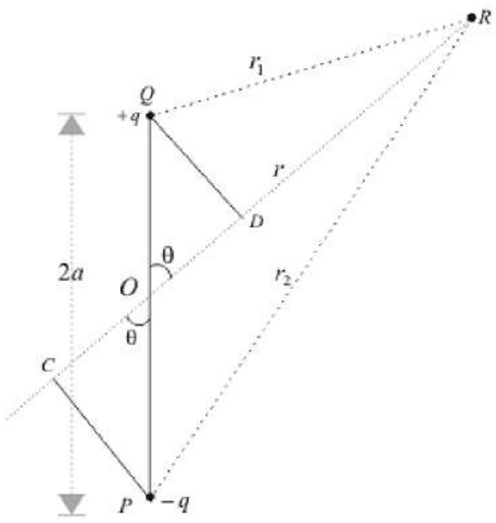
Q = the quantity of charge in coulombs

The potential difference is measured by an instrument called a voltmeter. The two terminals of a voltmeter are always connected parallel across the points whose potential is to be measured.

Electric Potential of a Dipole & a System of Charges

Potential due to an Electric Dipole

- The electric dipole is an arrangement that consists of two equal and opposite charges +q and -q separated by a small distance 2a.
- Electric dipole moment is represented by a vector p of magnitude 2qa and this vector points in the direction from -q to +q.
- To find electric potential due to a dipole consider charge -q is placed at point P and charge +q is placed at point Q as shown below in the figure.



A Dipole

- Since electric potential obeys the superposition principle so potential due to electric dipole as a whole would be sum of potential due to both the charges +q and -q. Thus

  $$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$$

  where r_1 and r_2 respectively are distance of charge +q and -q from point R.
- Now draw line PC perpandicular to RO and line QD perpandicular to RO as shown in figure.
- From triangle POC

  $\cos\theta = OC/OP = OC/a$

  therefore $OC = a\cos\theta$ similarly $OD = a\cos\theta$

  Now,

  $r_1 = QR \cong RD = OR - OD = r - a\cos\theta$

  $r_2 = PR \cong RC = OR + OC = r + a\cos\theta$

  $$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right)$$
- Since magnitude of dipole is $|\mathbf{p}| = 2qa$

  $$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2 - a^2\cos^2\theta}\right)$$
- If we consider the case where r>>a then

  $$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

  again since $p\cos\theta = \mathbf{p}\cdot\mathbf{\hat{r}}$ where, $\mathbf{\hat{r}}$ is the unit vector along the vector OR then electric potential of dipole is:

  $$V = \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}$$

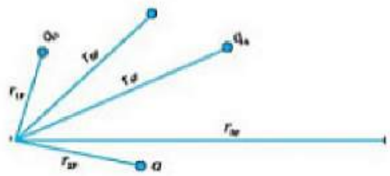
  for r>>a
- From above equation we can see that potential due to electric dipole is inversly proportional to r^2 not ad $1/r$ which is the case for potential due to single charge.

  Potential due to electric dipole does not only depends on r but also depends on angle between position vector **r** and dipole moment **p**.

Potential Due To A System Of Charges

- Consider a system of charges $q_1, q_2, \ldots, q_n$ with position vectors $r_1, r_2, \ldots, r_n$ relative to some origin. The potential V_1 at P due to the charge q_1 is:

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$



- where r_{1P} is the distance between q_1 and P.
- Similarly, the potential V_2 at P due to q_2 and due to q are given by

  $$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}}, \quad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}}$$

  where r_{2P} and r_{3P} are the distances of P from charges q_2 and q_3, respectively; and so on for the potential due to other charges.
- By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges

  $$V = V_1 + V_2 + \ldots + V_n$$

  $$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \ldots\ldots + \frac{q_n}{r_{nP}}\right)$$

- The electric field outside the shell is as if the **entire charge is concentrated at the centre.** Thus, the potential outside the shell is given by:

  $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad (r \geq R)$$

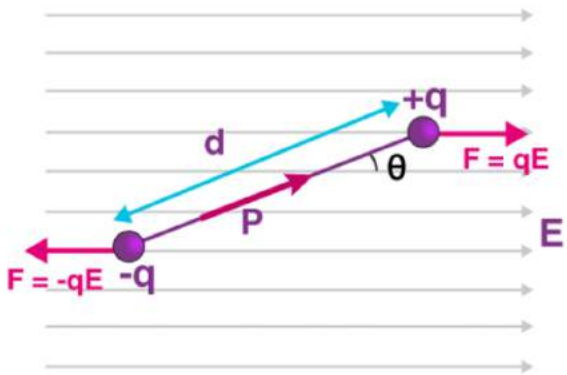
  where q is the total charge on the shell and R its radius.
- The electric field inside the shell is zero. This implies that potential is constant inside the shell (as **no work is done in moving a charge inside the shell**), and, therefore, equals its value at the surface, which is

  $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Potential Energy of a Dipole

How to Find Potential Energy of a Dipole?

- Consider a dipole with charges q1 = +q and q2 = -q placed in a uniform electric field as shown in the figure above. The charges are separated by a distance d and the magnitude of the electric field is E. The force experienced by the charges is given as –qE and +qE, as can be seen in the figure.



- As we know that, when a dipole is placed in a uniform electric field, both the charges as a whole do not experience any force, but it experiences a torque equal to τ which can be given as,

  $\tau = p \times E$

- This torque rotates the dipole unless it is placed parallel or anti-parallel to the field. If we apply an external and opposite torque, it neutralizes the effect of this torque given by τ_{ext} and it rotates the dipole from the angle Θ_0 to an angle Θ_1 at an infinitesimal angular speed without any angular acceleration.

  The amount of work done by the external torque can be given by:

$$W = \int_{\theta_0}^{\theta_1} \tau_{ext}(\theta) d\theta = \int_{\theta_0}^{\theta_1} pE \sin\theta \, d\theta$$

$$= pE(\cos\theta_0 - \cos\theta_1)$$

- As we know that the work done in bringing a system of charges from infinity to the given configuration is defined as the potential energy of the system, hence the potential energy U(Θ) can be associated to the inclination Θ of the dipole using the above relation:

  $U(\theta) = pE(\cos\theta_0 - \cos\theta_1)$

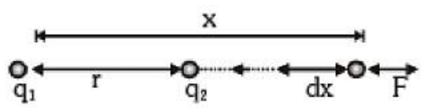
- From the above equation, we can see that the potential energy of a dipole placed in an external field is zero when the angle Θ is equal to 90° or when the dipole makes an angle of 90°. Considering the initial angle to be the angle at which the potential energy is zero, the potential energy of the system can be given as:

$$U(\theta) = pE(\cos\frac{\pi}{2} - \cos\theta) = -pE\cos\theta = -p.E$$

Electric Potential Energy of Charged Particles

Potential Energy of a System of two Charged Particles

Figure shows two + ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be expressed as work done in bringing charge q2 from infinity to the given separation from q_1.



It can be calculated as

$$W = \int_{\infty}^{r} \vec{F}.\vec{dx} = -\int_{\infty}^{r} \frac{kq_1q_2}{x^2}dx$$

[– ve sign shows that x is decreasing]

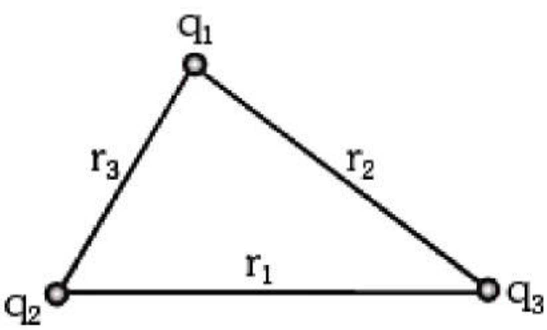
$$W = \frac{kq_1q_2}{r} = U \text{ [interaction energy]}$$

If the two charges are of opposite signs, then potential energy will be negative as

$$U = -\frac{kq_1q_2}{r}$$

Potential Energy for a System of Charged Particles

When more than two charged particles are there in a system, the interaction energy can be given as the sum of interaction energies of all the different possible pairs of particles. For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure.



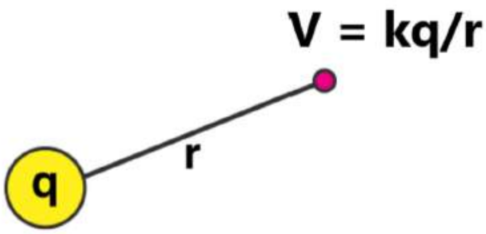
The total interaction energy of this system can be given as

$$U = \frac{kq_1q_2}{r_3} + \frac{kq_1q_3}{r_2} + \frac{kq_2q_3}{r_1}$$

Electrostatic Potential due to a Point Charge

What is Electrostatic Potential?

The electrostatic potential of a point charge Q is given by $V = kQ/r$.



Electric Potential

- Recall that the electric potential is defined as the potential energy per unit charge, i.e. $V = PE/q$
- The potential difference between two points ΔV is often called the voltage and is given by $\Delta V = VB - VA = \Delta PE/q$.
- The potential at an infinite distance is often taken to be zero.
- The case of the electric potential generated by a point charge is important because it is a case that is often encountered. A spherical sphere of charge creates an external field just like a point charge, for example.
- The equation for the electric potential due to a point charge is $V = kQ/r$, where k is a constant equal to 9.0×109 $N{\cdot}m^2/C^2$.
- The electric potential tells you how much potential energy a single point charge at a given location will have. The electric potential at a point is equal to the electric potential energy (measured in joules) of any charged particle at that location divided by the charge (measured in coulombs) of the particle.
- Since the charge of the test particle has been divided out, the electric potential is a "property" related only to the electric field itself and not the test particle. Another way of saying this is that because PE is dependent on q, the q in the above equation will cancel out, so V is not dependent on q.

- The potential difference between two points ΔV is often called the voltage and is given by:

$$\Delta V = V_B - V_A = \frac{\Delta PE}{q}$$

Electrostatic Potential Due to Point Charges

- Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere, see figure below) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider.
- Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q, and noting the connection between work and potential (W=−qΔV), it can be shown that the electric potential V of a point charge is
  V = kQ/r (point charge)
  where k is a constant equal to $9.0\times10^9\ N\cdot m^2/C^2$.
- The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas E for a point charge decreases with distance squared:
  $E = \frac{F}{q} = \frac{kQ}{r^2}$
- The electric potential is a scalar while the electric field is a vector. Note the symmetry between electric potential and gravitational potential – both drop off as a function of distance to the first power, while both the electric and gravitational fields drop off as a function of distance to the second power.

Superposition of Electric Potential

- To find the total electric potential due to a system of point charges, one adds the individual voltages as numbers. The electric potential V is a scalar and has no direction, whereas the electric field E is a vector.
- To find the voltage due to a combination of point charges, you add the individual voltages as numbers. So for example, in the electric potential at point L is the sum of the potential contributions from charges Q1, Q2, Q3, Q4, and Q5 so that $V_L = k[\frac{Q_1}{d_1} + \frac{Q_2}{d_2} + \frac{Q_3}{d_3} + \frac{Q_4}{d_4} + \frac{Q_5}{d_5}]$.
- To find the total electric field, you must add the individual fields as vectors, taking magnitude and direction into account. This is consistent

with the fact that V is closely associated with energy, a scalar, whereas E is closely associated with force, a vector.

- The summing of all voltage contributions to find the total potential field is called the superposition of electric potential. It is much easier to sum scalars than vectors, so often the preferred method for solving problems with electric fields involves the summing of voltages.

Key Terms

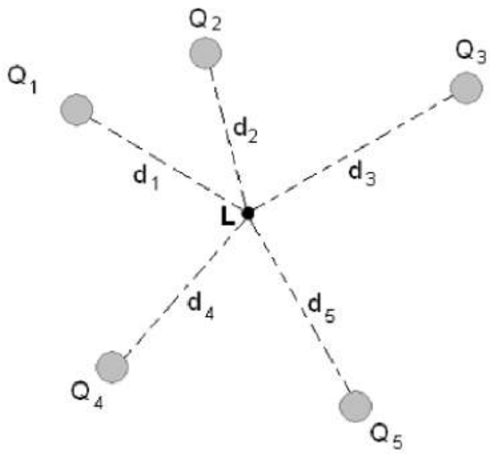
Vector: A directed quantity, one with both magnitude and direction; the between two points.
Scalar: A quantity that has magnitude but not direction; compare vector.
Superposition: The summing of two or more field contributions occupying the same space.

- We've seen that the electric potential is defined as the amount of potential energy per unit charge a test particle has at a given location in an electric field, i.e.

  $V = PE/q$

- We've also seen that the electric potential due to a point charge is $V = kQ/r$
  where k is a constant equal to $9.0\times10^{9}\ N\cdot m^2/C^2$. The equation for the electric potential of a point charge looks similar to the equation for the electric field generated for a point particle:
  $E = \frac{F}{q} = \frac{kQ}{r^2}$

With the difference that the electric field drops off with the square of the distance while the potential drops off linearly with distance. This is analogous to the relationship between the gravitational field and the gravitational potential.



Superposition of Electric Potential: The electric potential at point L is the sum of voltages from each point charge (scalars)

- Recall that the electric potential V is a scalar and has no direction, whereas the electric field E is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. So for example, in the figure above the electric potential at point L is the sum of the potential contributions from charges Q_1, Q_2, Q_3, Q_4, and Q_5 so that

$$V_L = k[\frac{Q_1}{d_1} + \frac{Q_2}{d_2} + \frac{Q_3}{d_3} + \frac{Q_4}{d_4} + \frac{Q_5}{d_5}]$$

- To find the total electric field, you must add the individual fields as vectors, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas E is closely associated with force, a vector.
- The summing of all voltage contributions to find the total potential field is called the superposition of electric potential.
- Summing voltages rather than summing the electric simplifies calculations significantly, since addition of potential scalar fields is much easier than addition of the electric vector fields. Note that there are cases where you might need to sum potential contributions from sources other than point charges; however, that is beyond the scope of this section.

Example.1 What Voltage Is Produced by a Small Charge on a Metal Sphere?
Solution.

- Charges in static electricity are typically in the nano coulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a −3.00nC static charge?

- As we have discussed in Electric Charge and Electric Field, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation V=kQ/r.
- Entering known values into the expression for the potential of a point charge, we obtain

$$V = k\frac{Q}{r}$$

$$= \left(8.99 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}\right)\left(\frac{-3.00 \times 10^{\ 9}\ \mathrm{C}}{5.00 \times 10^{\ 2}\ \mathrm{m}}\right)$$

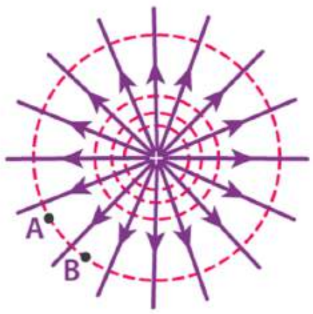
= –539 V.

- **Discussion:** The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower—more negative—than at larger distances. Conversely, a negative charge would be repelled, as expected.

Equipotential Surfaces

What is Equipotential Surface?

The surface which is the locus of all points which are at the same potential is known as the equipotential surface. No work is required to move a charge from one point to another on the equipotential surface.



Equipotential Surface

- In other words, any surface with the same electric potential at every point is termed as an equipotential surface.

Equipotential Points

- If the points in an electric field are all at the same electric potential, then they are known as the equipotential points. If these points are connected by a line or a curve, it is known as an equipotential line.

- If such points lie on a surface, it is called an equipotential surface. Further, if these points are distributed throughout a space or a volume, it is known as an equipotential volume.

Work Done in Equipotential Surface

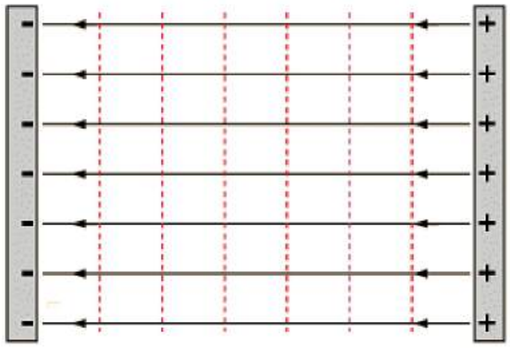
- The work done in moving a charge between two points in an equipotential surface is zero. If a point charge is moved from point V_A to V_B, in an equipotential surface, then the work done in moving the charge is given by:
  $W = q_0(V_A - V_B)$
  As $V_A - V_B$ is equal to zero, the total work done is $W = 0$.

Properties of Equipotential Surface

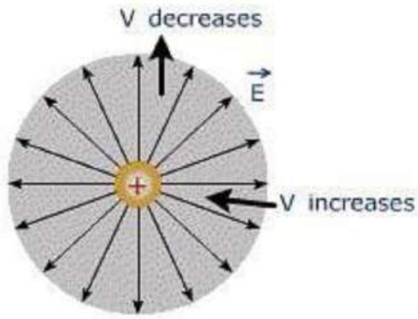
1. The electric field is always perpendicular to an equipotential surface.
2. Two equipotential surfaces can never intersect.
3. For a point charge, the equipotential surfaces are concentric spherical shells.
4. For a uniform electric field, the equipotential surfaces are planes normal to the x-axis
5. The direction of the equipotential surface is from high potential to low potential.
6. Inside a hollow charged spherical conductor the potential is constant. This can be treated as equipotential volume. No work is required to move a charge from the centre to the surface.
7. For an isolated point charge, the equipotential surface is a sphere. i.e. concentric spheres around the point charge are different equipotential surfaces.
8. In a uniform electric field, any plane normal to the field direction is an equipotential surface.
9. The spacing between equipotential surfaces enables us to identify regions of a strong and weak field i.e. $E = -dV/dr \Rightarrow E \propto 1/dr$

Equipotential Lines in a Constant Field

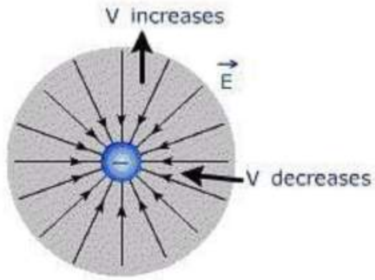
- In a conducting plate-like in a capacitor, the electric field lines are perpendicular to the plates and the equipotential lines are parallel to the plates.



- The illustration below shows the electric field of a positive point charge The electric field is fixed away from the charge and potential is positive at any set distance from the charge. If the charge moves in the direction of the electric field it will move towards the lower values of potential.
- If the charge moves towards the direction opposite to that of electric field we move towards the higher values of potential.



- The illustration below shows the electric field of a point negative charge.



- Limited distance is negative at any point from the charge. If the charge moves toward the direction of the electric field and in the direction of decreasing U thus, becoming move negative. If a charge moves in the direction opposite to electric field, the increasing value of V thus, become less negative.

- Hence, the moving with the direction of electric field means moving in the direction of decreasing V and moving against the direction of electric field means moving in the direction of increasing V. Therefore, potential difference canbe expressed in terms of electric field as:

$$V_a - V_b = -\int_b^a \vec{E} \cdot \vec{dl}$$

- Electric field (E) as a function of potential can be expressed as

$$\vec{E_r} = -\frac{dv}{dr}$$

where Er is the component of electric field along the direction of dv/dr is known as the potential gradient and the negative sign infers that electric field acts in a direction of decrease of potential.

- The expression above indicates that E is not certainly zero if V is zero.

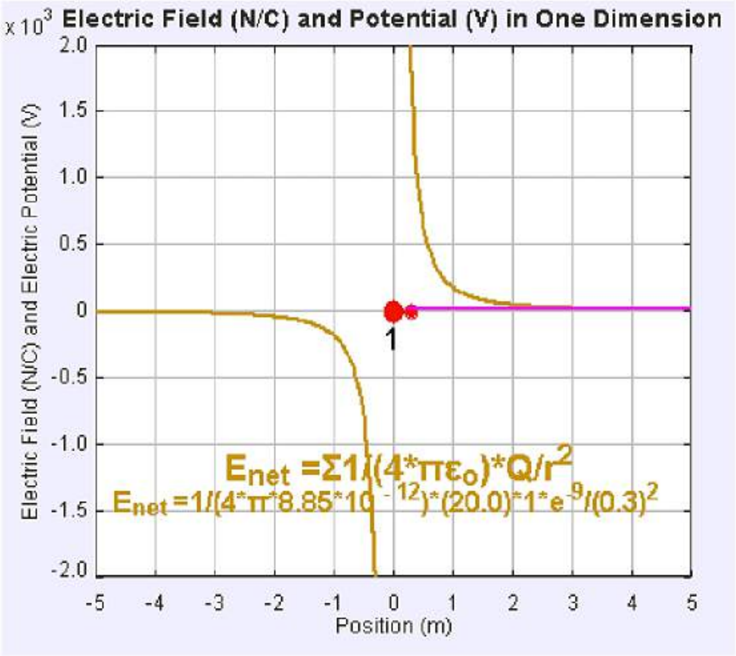
Relation between Electric Field, Potential & Potential Energy of a System of Charges

Introduction

- **Electric Potential:** The potential energy per unit charge at a point in a static electric field; voltage.
- **Electric Field:** A region of space around a charged particle, or between two voltages; it exerts a force on charged objects in its vicinity.
- For a uniform field, the relationship between electric field (E), potential difference between points A and B (Δ), and distance between points A and B (d) is: $E = -\Delta\phi/d$
- If the field is not uniform, calculus is required to solve. Potential is a property of the field that describes the action of the field upon an object.

Relation Between Electric Field and Electric Potential

The relationship between electric potential and field is similar to that between gravitational potential and field in that the potential is a property of the field describing the action of the field upon an object.



Electric Field and Potential in one Dimension

- The presence of an electric field around the static point charge (large red dot) creates a potential difference, causing the test charge (small red dot) to experience a force and move.
- The electric field is like any other vector field—it exerts a force based on a stimulus, and has units of force times inverse stimulus. In the case of an electric field the stimulus is charge, and thus the units are NC-1. In other words, the electric field is a measure of force per unit charge.
- The electric potential at a point is the quotient of the potential energy of any charged particle at that location divided by the charge of that particle. Its units are JC-1. Thus, the electric potential is a measure of energy per unit charge.
- In terms of units, electric potential and charge are closely related. They share a common factor of inverse Coulombs (C-1), while force and energy only differ by a factor of distance (energy is the product of force times distance).
- Thus, for a uniform field, the relationship between electric field (E), potential difference between points A and B (Δ), and distance between points A and B (d) is:

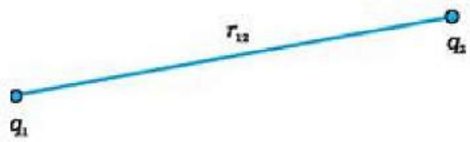
  $E = -\Delta\phi/d$
- The -1 coefficient arises from repulsion of positive charges: a positive charge will be pushed away from the positively charged plate, and towards a location of higher-voltage.

- The above equation is an algebraic relationship for a uniform field. In a more pure sense, without assuming field uniformity, electric field is the gradient of the electric potential in the direction of x:
  Ex = − dx/dV .
- This can be derived from basic principles. Given that ΔP=W (change in the energy of a charge equals work done on that charge), an application of the law of conservation of energy, we can replace ΔP and W with other terms. ΔP can be substituted for its definition as the product of charge (q) and the differential of potential (dV). We can then replace W with its definition as the product of q, electric field (E), and the differential of distance in the x direction (dx):
  $qdV = -qE_x dx$.

Potential Energy of A System of Charges

- Consider the charges q_1 and q_2 initially at infinity and determine the work done by an external agency to bring the charges to the given locations.
- Suppose, charge q_1 is brought from infinity to the point r_1. There is no external field against which work needs to be done, so work done in bringing q1 from infinity to r_1 is zero. This charge produces a potential in space given by

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$



- where r1P is the distance of a point P in space from the location of q_1.
- From the definition of potential, work done in bringing charge q_2 from infinity to the point r_2 is q times the potential at r_2 due to q_1:

$$\text{work done on } q_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

  where r_{12} is the distance between points 1 and 2.
- If $q_1 q_2 > 0$, Potential energy is positive. For unlike charges ($q_1\, q_2 < 0$), the electrostatic force is attractive.
- Potential energy of a system of three charges q1, qand q located at r_1, r_2, r, respectively. To bring q first from infinity to r_1, no work is required. Next bring q2 from infinity to r_2. As before, work done in this step is

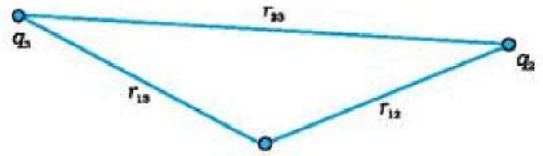
$$q_2 V_1(\mathbf{r}_2) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

The charges q_1 and q_2 produce a potential, which at any point P is given by

$$V_{1,2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}}\right)$$

Work done next in bringing q_3 from infinity to the point r_3 is q3 $V_{1,2}$ at $\mathbf{r}_3$

$$q_3 V_{1,2}(\mathbf{r}_3) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$



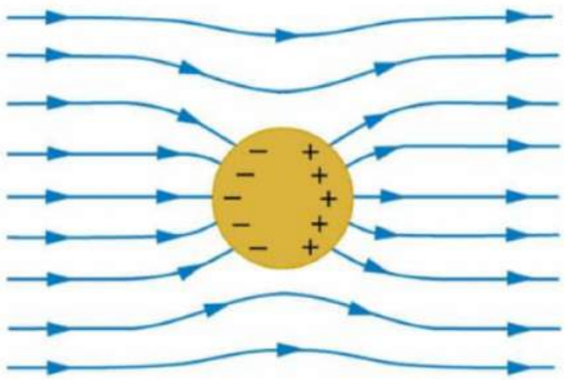
The total work done in assembling the charges at the given locations is obtained by adding the work done in different steps,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Electrostatics of Conductors

Electrostatic Properties of Conductors

- Conductors contain mobile charge carriers. In metallic conductors, these charge carriers are electrons. In a metal, the outer (valence) electrons part away from their atoms and are free to move. These electrons are free within the metal but not free to leave the metal.



- In an external electric field, they drift against the direction of the field. The positive ions made up of the nuclei and the bound electrons remain held in their fixed positions. In electrolytic conductors, the charge carriers are both positive and negative ions. Some of the important points about the electrostatic properties of a conductor are as follows:

1. The electrostatic field is zero inside a conductor

- In the static condition, whether a conductor is neutral or charged, the electric field inside the conductor is zero everywhere. This is one of the defining properties of a conductor.
- We know that a conductor contains free electrons which, in the presence of an electric field, experience a drift or a force. Inside the conductor, the electrons distribute themselves in such a way that the final electric field at all points inside the conductor is zero.

2. Electrostatic field lines are normal to the surface at every point in a charged conductor

- We can say, if the electric field lines were not normal at the surface, a component of the electric field would have been present along the surface of a conductor in the static condition.
- Thus, free charges moving on the surface would also have experienced some force leading to their motion. But, this does not happen. Since there are no tangential components, the forces have to be normal to the surface.

3. In static conditions, the interior of the conductor contains no excess charge

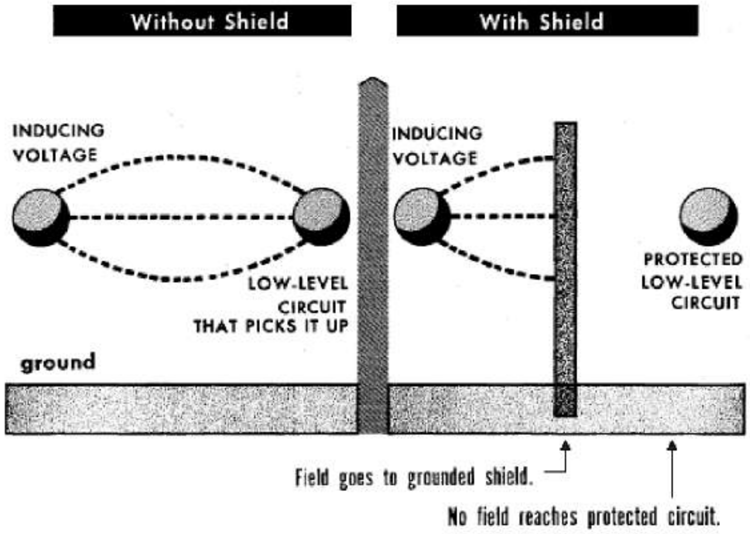
- We know that any neutral conductor contains an equal amount of positive and negative charges, at every point. This holds true even in an infinitesimally small element of volume or surface area.
- From the Gauss's law, we can say that in the case of a charged conductor, the excess charges are present only on the surface.
- Let us consider an arbitrary volume element of the conductor, which we denote as 'v' and for the closed surface bounding the volume element, the electrostatic field is zero. Thus, the total electric flux through S is zero. So, from the Gauss law, it follows that the net charge enclosed by the surface element is zero.
- As we go on decreasing the size of the volume and the surface element, at a point we can say that when the element is vanishingly small, it denotes any point in the conductor. So the net charge at any point inside the conductor is always zero and the excess charges reside at the surface.

4. Constant electrostatic potential throughout the volume of the conductor

- The electrostatic potential at any point throughout the volume of the conductor is always constant and the value of the electrostatic potential at the surface is equal to that at any point inside the volume.

5. Electrostatic Shielding

- The third kind of shielding is a protection against electric induction, commonly called electrostatic induction. (Electrostatic induction is really a misnomer; this induction is better called electric induction, because it depends upon the continuous fluctuation of the charges induced.)

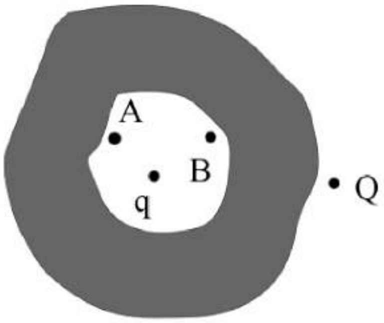


Electrostatic Shielding

- Electric shielding consists of interposing a grounded shield between the interfering voltage and the low-level circuit that might pick up the electric field. The voltage induced is immediately carried off to ground, and the electric field is prevented from reaching the circuit that is shielded.
- Unlike either form of protection against magnetic fields, shielding against electric fields can be almost 100% effective. All that is necessary is to insure that no path is left through which the electric field can pass. For this reason, much more attention is given to the prevention of induction by magnetic fields.

Solved Example

Assertion: A point charge is placed inside a cavity of the conductor as shown. Another point charge Q is placed outside the conductor as shown. Now as the point charge Q is pushed away from the conductor, the potential difference ($V_A - V_B$) between two points A and B within the cavity of sphere remains constant.
Reason: The electric field due to charge on the outer surface of the conductor and outside the conductor is zero at all points inside the conductor



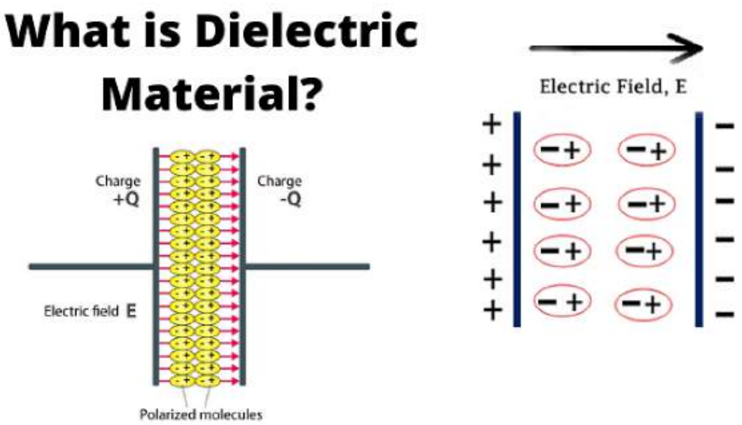
A. Both Assertion and Reason are correct and the Reason is the correct explanation for Assertion
B. Both Assertion and Reason are correct and the Reason is not the correct explanation for Assertion
C. Assertion is correct but Reason is incorrect
D. Assertion is incorrect but Reason is correct

Solution:
Option A. As we know that that the electric field inside the conductor is zero, so the field inside the conductor is constant. Ther efore the potential between point A and B will remain constant.

Effect of Dielectric on Capacitance

What are Dielectrics?

- Dielectrics are basically insulating and non-conducting substances. They are bad conductors of electric current. Dielectrics are capable of holding electrostatic charges while emitting minimal energy. This energy is usually in the form of heat.



- The common examples of dielectrics include mica, plastics, porcelain, metal oxides and glass etc. It is important for you to note that dry air is also a dielectric.

What is the Dielectric Constant?

When we put a dielectric slab in between two plates of a parallel plate capacitor, the ratio of the applied electric field strength to the strength of the reduced value of electric field capacitor is called the dielectric constant. It is given as

$K = E_o/E$

E_o is greater than or equal to E, where E_o is the field with the slab and E is the field without it. The larger the dielectric constant, the more charge can be stored. Completely filling the space between capacitor plates with a dielectric, increases the capacitance by a factor of the dielectric constant:

$C = KC_o$,

where C_o is the capacitance with no slab between the plates. This is all about a quick recap. Now let us move ahead and see what effect dielectrics have on the capacitance.

Force Between the Plates of a Capacitor

In a capacitor as plates carry equal and opposite charges, there is a force of attraction between the plates. To calculate this force, we use the fact that the electric field is conservative and in a conservative field $F = -\frac{dU}{dx}$.

In the case of parallel plate capacitor:

$U = \frac{q^2}{2C} = \frac{1}{2}\frac{q^2 x}{\varepsilon_0 A}$ [as $C = \frac{\varepsilon_0 A}{x}$]

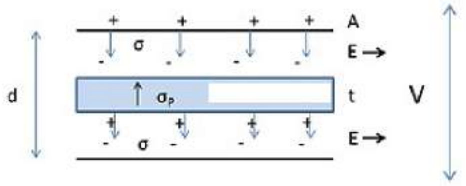
So, $F = -\frac{d}{dx}\left[\frac{q^2}{2\varepsilon_0 A}x\right] = \frac{-1}{2}\frac{q^2}{\varepsilon_0 A}$

The negative sign implies that the force is attractive.

Effect of Dielectric on Capacitance

We usually place dielectrics between the two plates of parallel plate capacitors. They can fully or partially occupy the region between the plates. When we place the dielectric between the two plates of a parallel plate capacitor, the electric field polarises it.

The surface charge densities are σ_p and $-\sigma_p$. When we place the dielectric fully between the two plates of a capacitor, then it's dielectric constant increases from its vacuum value.



Dielectric inside a parallel plate capacitor

The electric field inside a capacitor is as follows:

$$E = \frac{\sigma - \sigma_E}{\varepsilon_0}$$

Hence we have:

$$V = \frac{\sigma d}{\varepsilon_0 k} = \frac{Qd}{A\varepsilon_0 k}$$

Therefore:

$$C = \frac{Q}{V} = \frac{A\varepsilon_0 k}{d} = \frac{A\varepsilon}{d}$$

Ɛ is the permittivity of the substance. The potential difference between the plates is given by

$$V = Ed = \frac{\sigma - \sigma_p}{\varepsilon_0} d$$

For linear dielectrics:

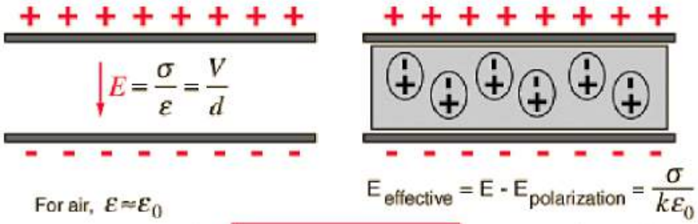
$$\sigma - \sigma_p = \frac{\sigma}{k}$$

Where k is a dielectric constant of the substance, K = 1.

$$K = \frac{\varepsilon}{\varepsilon_0}$$

How does the dielectric increase the capacitance of a capacitor?

The electric field between the plates of parallel plate capacitor is directly proportional to capacitance C of the capacitor. The strength of the electric field is reduced due to the presence of dielectric. If the total charge on the plates is kept constant, then the potential difference is reduced across the capacitor plates. In this way, dielectric increases the capacitance of the capacitor.



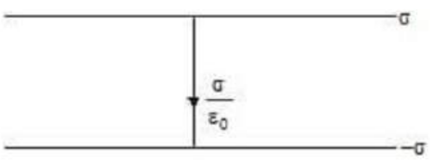
$$C = \frac{\varepsilon_0 A}{d}$$

The capacitance is increased by the factor k.

$$C = \frac{k\varepsilon_0 A}{d}$$

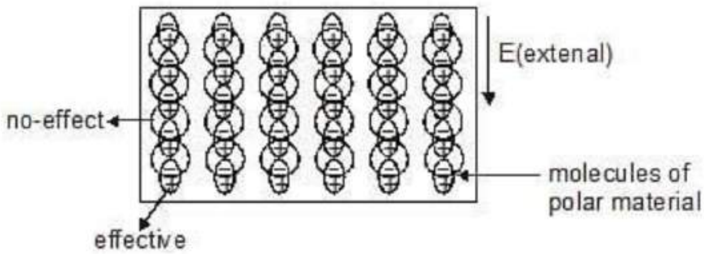
Capacitors with Dielectric

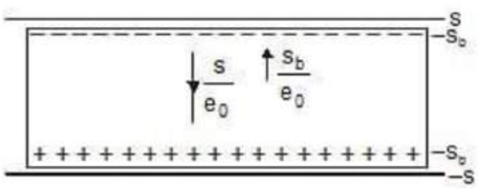
(i) In absence of dielectric



$E = \frac{\sigma}{\varepsilon_0}$

(ii) When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.





s_b = induced charge density (called bound charge because it is not due to free electrons).

* For polar molecules dipole moment $\neq 0$
* For non-polar molecules dipole moment = 0

(iii) Capacitance in the presence of dielectric

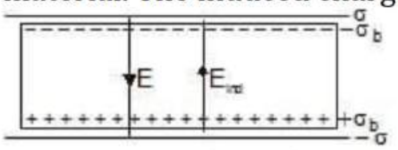
$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{K\varepsilon_0}.d} = \frac{AK\varepsilon_0}{d} = \frac{AK\varepsilon_0}{d}$$

Here capacitance is increased by a factor K.

$$C = \frac{AK\varepsilon_0}{d}$$

(iv) Polarisation of material:

When a non-polar substance is placed in an electric field then dipole moment is induced in the molecule. This induction of dipole moment is called polarisation of material. The induced charge also produces electric field.



σ_b = induced (bound) charge density.

$$E_{in} = E - E_{ind} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0}$$

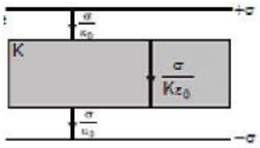
It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by e_r or k.

$$E_{in} = \frac{\sigma}{K\varepsilon_0} \Rightarrow \sigma_b = \sigma\left(1 - \frac{1}{K}\right)$$

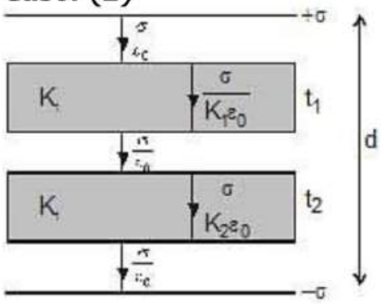
(v) If the medium is not filled between the plates completely then electric filed will be as shown in figure.

Case: (1)

The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other.



Case: (2)



so potential difference between plates

$$v = \frac{\sigma}{\varepsilon_0}[d - t_1 - t_2] + \frac{\sigma}{k_1\varepsilon_0}t_1 + \frac{\sigma}{k_2\varepsilon_0}t_2$$

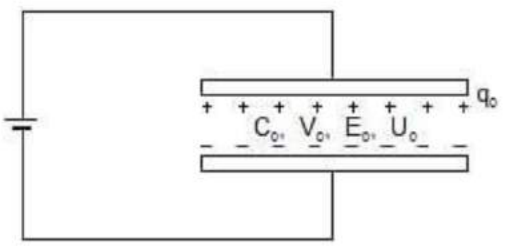
so equivalent capacitance

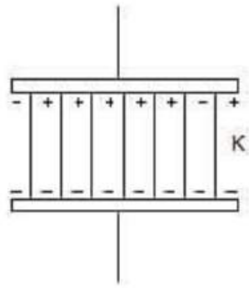
$$C = \frac{Q}{v}$$

$$C = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[d - t_1 - t_2 + \frac{t_1}{k_1} + \frac{t_2}{k_2}\right]}$$

$$C = \frac{A\varepsilon_0}{d - t_1\left(1 - \frac{1}{k_1}\right) - t_2\left(1 - \frac{1}{k_2}\right)}$$

10.1 Introduction of a Dielectric slab of dielectric constant K between the plates
(a) When the battery is disconnected





Let q_0, C_0, V_0, E_0 and U_0 represents the charge, capacity, potential difference, electric field and energy associated with charged air capacitor respectively. With the introduction of a dielectric slab of dielectric constant K between the plates and the battery disconnected.

(i) Charge remains constant, i.e., $q = q_0$, as in an isolated system charge is conserved.

(ii) Capacity increases, i.e., $C = KC_0$, as by the presence of a dielectric capacity becomes K times.

(iii) Potential difference between the plates decreases, i.e., $V = \left(\frac{V_0}{K}\right)$, as

$$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K} \quad [\because q = q_0 \text{ and } C = KC_0]$$

(iv) Field between the plates decreases, i.e., $E = \frac{E_0}{K}$, as

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K} \qquad \left[\text{as } V = \frac{V_0}{K}\right]$$

and $E_0 = \frac{V_0}{d}$

(v) Energy stored in the capacitor decreases i.e.

$U = \left(\frac{U_0}{K}\right)$, as

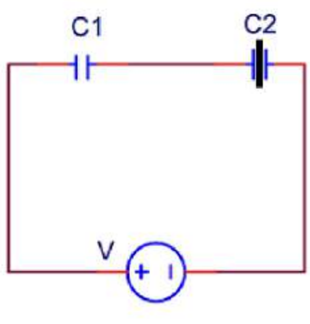
$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{V_0}{K}$ (as $q = q_0$ and $C = KC_0$]

(b) When the battery remains connected (potential is held constant)

(i) Potential difference remains constant, i.e., $V = V_0$, as battery is a source of constant potential difference.
(ii) Capacity increases, i.e., $C = KC_0$, as by presence of a dielectric capacity becomes K times.
(iii) Charge on capacitor increases, i.e., $q = Kq_0$, as
$q = CV = (KC_0)\,V = Kq_0$ [$\because\ q_0 = C_0V$]
(iv) Electric field remains unchanged, i.e., $E = E_0$, as
$E = \frac{V}{d} = \frac{V_0}{d} = E_0$ [as $V = V_0$ and $\frac{V_0}{d} = E_0$]
(v) Energy stored in the capacitor increases, i.e., $U = KU_0$, as
$U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2 = \frac{1}{2}KU_0$ [as $C = KC_0$ and $U_0 = \frac{1}{2}C_0V_0^2$]

Solved Examples

Example.1 Assertion: In a circuit where two capacitors with capacitance C_1 and C_2 are connected in series with C_1 followed by C_2. A slab is inserted in C_2. The potential difference across C_2 will decrease.
Reason: The current flows in the clockwise direction, after the introduction of the slab.



(a) Both the statements are correct and the reason is the correct explanation of the assertion.
(b) They both are correct but the reason is not the correct explanation of the assertion.

(c) The assertion is correct but the reason is not.
(d) The reason is correct but the assertion is not.

Solution: B) Upon inserting the slab, the value of C_2 will increase to KC_2. In series, V distributes in the inverse ratio of capacitance. As C_2 increases, its potential will decrease. As C_2 increases, the positive charge on both the capacitors will have to increase. Thus the current will flow in the clockwise direction.

Example.2 A parallel plate air capacitor is made using two square plates each of side 0.2 m, spaced 1 cm apart. It is connected to a 50V battery.
(a) What is the capacitance?
(b) What is the charge on each plate?
(c) What is the energy stored in the capacitor?
(d) What is the electric field between the plates?
(e) If the battery is disconnected and then the plates are pulled apart to a separation of 2 cm, what are the answers to the above parts?

Sol. (a) $C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}) \times 0.2 \times 0.2}{0.01} = 3.54 \times 10^{-5}\ \mu F$

(b) $Q_0 = C_0V_0 = 3.54 \times 10^{-5} \times 50 = 1.77 \times 10^{-3}$ mC

(c) $U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2} \times (3.54 \times 10^{-11})(50)^2 = 4.42 \times 10^{-8} J$

(d) $E_0 = \frac{V_0}{d} = \frac{50}{0.01} = 5000\ V/m$

(e) If the battery is disconnected the charge on the capacitor plates remains constant while the potential difference between the plates can change.

$$C = \frac{\varepsilon_0 A}{d} = \frac{C_0}{2} = 1.77 \times 10^{-5} \mu F$$

$Q = Q_0 = 1.77 \times 10^{-3}$ mC

$$V = \frac{Q}{C} = \frac{Q_0}{C_0/2} = 2V_0$$

$$U = \frac{1}{2}CV^2 = C_0V_0^2 = 8.84 \times 10^{-8} J$$

$$E = \frac{V}{d} = \frac{2V_0}{2d_0} = E_0 = 5000\ V/m$$

Example.3 In the last illustration, suppose that the battery is kept connected while the plates are pulled apart. What are the answers to the parts (a), (b), (c) and (d) in that case?
Sol. If the battery is kept connected, the potential difference across the capacitor plates always remains equal to the emf of battery and hence is constant.
$V = V_0 = 50V$

$$C = \frac{\varepsilon_0 A}{d'} = \frac{\varepsilon_0 A}{2d} = \frac{C_0}{2} = 1.77 \times 10^{-5}\ \mu F$$

$$Q = CV = \frac{C_0 V_0}{2} = \frac{Q_0}{2} = 8.85 \times 10^{-4}\ \mu C$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{C_0}{2}\right)V_0^2 = \frac{U_0}{2} = 2.21 \times 10^{-8}\ J$$

$$E = \frac{V}{d} = \frac{V_0}{2d_0} = \frac{E_0}{2} = 2500\ V/m$$

Example.4 A parallel plate capacitor has plates of area 4 m² separated by distance of 0.5 mm. The capacitor is connected across a cell of emf 100 V.
(a) Find the capacitance, charge and energy stored in the capacitor.
(b) A dielectric slab of thickness 0.5 mm is inserted inside this capacitor after it has been disconnected from the cell. Find the answers to part (a) if K = 3.

Sol. (a) $C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 4}{0.5 \times 10^{-3}} = 7.08 \times 10^{-2}\ \mu F$

$Q_0 = C_0 V_0 = (7.08 \times 10^{-2} \times 100)$ mC $= 7.08$ mC

$U_0 = \frac{1}{2}C_0 V_0^2 = 3.54 \times 10^{-4}\ J$

(b) As the cell has been disconnected

$Q = Q_0$

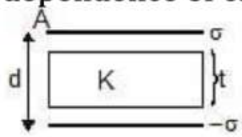
$C = \frac{K\varepsilon_0 A}{d} = KC_0 = 0.2124\ \mu F$

$V = \frac{Q}{C} = \frac{Q_0}{KC_0} = \frac{V_0}{K} = \frac{100}{3} V$

$U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K} = 118 \times 10^{-6}$ J

Example.5 If a dielectric slab of thickness t and area A is inserted in between the plates of a parallel plate capacitor of plate area A and distance between the plates d (d > t) then find out capacitance of system. What do you predict about the dependence of capacitance on location of slab ?

Sol.



$C = \frac{Q}{V} = \frac{\sigma A}{V}$

$V = \frac{\sigma t_1}{\varepsilon_0} + \frac{\sigma t}{K\varepsilon_0} + \frac{\sigma t_2}{\varepsilon_0}$ $(\because\ t_1\ t_2 = d - t)$

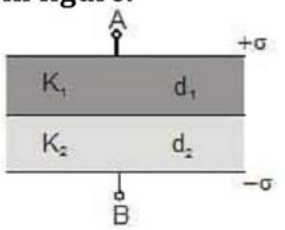
$= \frac{\sigma}{\varepsilon_0}[t_1 + t_2 + t/k]$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{k}\right] = \frac{Q}{C} = \frac{\sigma A}{C} \quad \Rightarrow \quad C = \frac{\varepsilon_0 A}{d - t + t/K}$$

* Capacitance does not depend upon the position of dielectric (it can be shifted up or down & still capacitance does not change).
* If the slab is of metal then

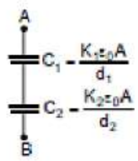
$$C = \frac{A\varepsilon_0}{d - t}$$

Example.6 Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of thickness d_1 and d_2 and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



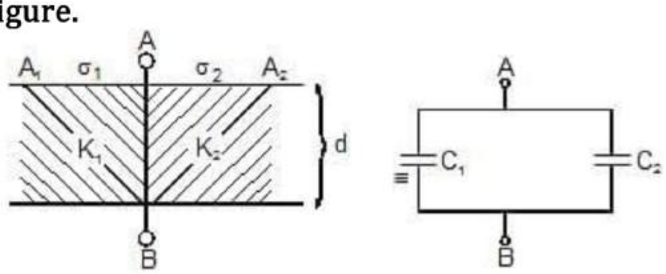
Sol. $C = \frac{\sigma A}{V}$; $V = E_1 d_1 + E_2 d_2 = \frac{\sigma d_1}{K_1\varepsilon_0} + \frac{\sigma d_2}{K_2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2}\right)$

$$\therefore \quad C = \frac{A\varepsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \Rightarrow \quad \frac{1}{C} = \frac{d_1}{AK_1\varepsilon_0} + \frac{d_2}{AK_2\varepsilon_0}$$



This formula suggests that the system between A and B can be considered as series combination of two capacitors.

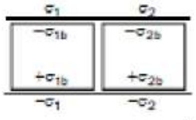
Example.7 Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of area A_1 and A_2 and each of thickness d are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



Sol.

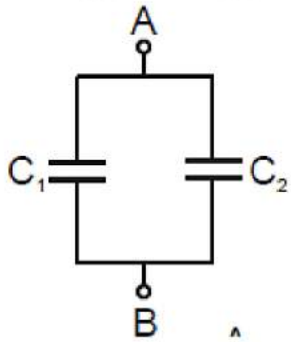
$C_1 = \frac{A_1K_1\varepsilon_0}{d}$, $C_2 = \frac{A_2K_2\varepsilon_0}{d}$

$$E_1 = \frac{V}{d} = \frac{\sigma_1}{K_1\varepsilon_0} \; ; \quad E_2 = \frac{V}{d} = \frac{\sigma_2}{K_2\varepsilon_0}$$



$$\sigma_1 = \frac{K_1\varepsilon_0 V}{d} \quad \sigma_2 = \frac{K_2\varepsilon_0 V}{d}$$

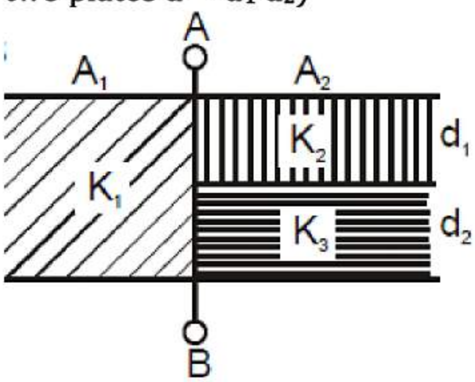
$$C = \frac{Q_1 + Q_2}{V} = \frac{\sigma_1 A_1 + \sigma_2 A_2}{V} = \frac{K_1\varepsilon_0 A_1}{d} + \frac{K_2\varepsilon_0 A_2}{d}$$



The combination is equivalent to :

Therefore, $C = C_1 \; C_2$

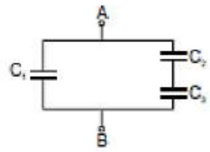
Example.8 Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d_1 K_2 of area A_2 and thickness d_2 and K_3 of area A_2 and thickness d_3 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d = d_1 \; d_2$)



Sol. It is equivalent to

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

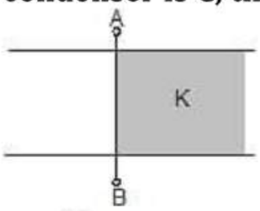
$$C=\frac{A_1K_1\varepsilon_0}{d_1+d_2}+\frac{\frac{A_2K_2\varepsilon_0}{d_1}\cdot\frac{A_2K_3\varepsilon_0}{d_2}}{\frac{A_2K_2\varepsilon_0}{d_1}+\frac{A_2K_3\varepsilon_0}{d_2}}$$



$$=\frac{A_1K_1\varepsilon_0}{d_1+d_2}+\frac{A_2^2K_2K_3\varepsilon_0^2}{A_2K_2\varepsilon_0d_2+A_2K_3\varepsilon_0d_1}$$

$$=\frac{A_1K_1\varepsilon_0}{d_1+d_2}+\frac{A_2K_2K_3\varepsilon_0}{K_2d_2+K_3d_1}$$

Example.9 A dielectric of constant K is slipπεd between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be



(A) $\frac{C}{2}$

(B) $\frac{C}{2K}$

(C) $\frac{C}{2}[1+K]$

(D) $\frac{2[1+K]}{C}$

Sol. This system is equivalent to two capacitors in parallel with area of each plate.

Capacitor & Capacitance

What is a Capacitor?

A capacitor is a two-terminal electrical device that possesses the ability to store energy in the form of an electric charge. It consists of two electrical conductors that are separated by a distance.

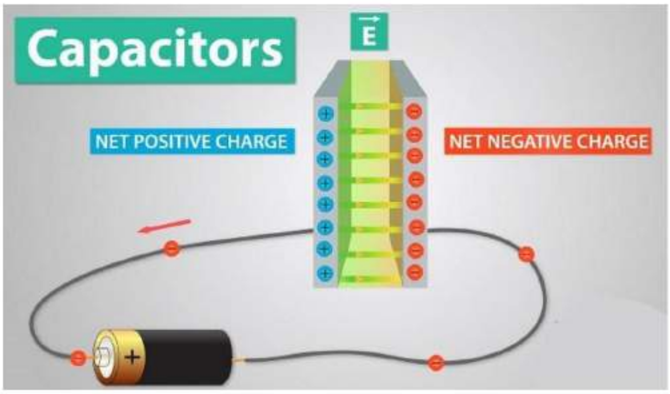
Different Types of Capacitor

- The space between the conductors may be filled by vacuum or with an insulating material known as a dielectric. The ability of the capacitor to store charges is known as capacitance.
- Capacitors store energy by holding apart pairs of opposite charges. The simplest design for a capacitor is a parallel plate, which consists of two metal plates with a gap between them. But, there are different types of capacitors manufactured in many forms, styles, lengths, girths, and many materials.

How Does a Capacitor Work?

- For demonstration, let us consider the most basic structure of a capacitor – the parallel plate capacitor. It consists of two parallel plates separated by a dielectric. When we connect a DC voltage source across the capacitor, one plate is connected to the positive end (plate I) and the other plate to the negative end (plate II).

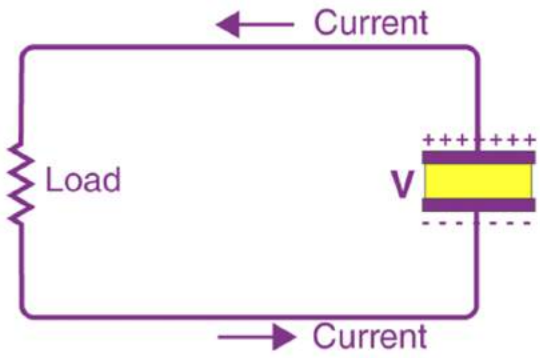
When the potential of the battery is applied across the capacitor, plate I become positive with respect to plate II. At the steady-state condition, the current tries to flow through the capacitor from its positive plate to its negative plate. But it is unable to flow due to the separation of these with an insulating material.



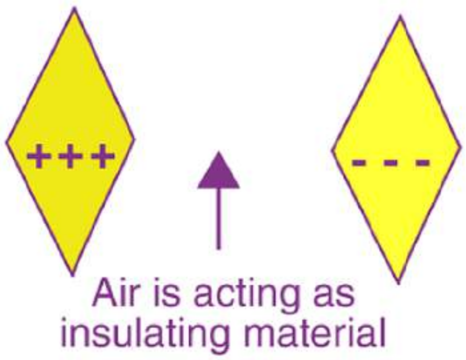
How Does a Capacitor Work?

- An electric field appears across the capacitor. The positive plate (plate I) accumulates positive charges from the battery, and the negative plate (plate II) will accumulate negative charges from the battery. After a point, the capacitor holds the maximum amount of charge as per its capacitance with respect to this voltage. This time span is called the charging time of the capacitor.
- When the battery is removed from the capacitor, the two plates hold a negative and positive charge for a certain time. Thus, the capacitor acts as a source of electrical energy.

- If these plates are connected to a load, the current flows to the load from Plate I to Plate II until all the charges are dissipated from both plates. This time span is known as the discharging time of the capacitor.



How do you Determine the Value of Capacitance?



The conducting plates have some charges q_1 and q_2 (Usually if one plate has +q the other has –q charge). The electric field in the region between the plates depends on the charge given to the conducting plates. We also know that potential difference (V) is directly proportional to the electric field hence we can say,

$$Q \propto V$$

$$Q = CV$$

$$C = \frac{Q}{V}$$

- This constant of proportionality is known as the capacitance of the capacitor.

- Capacitance is the ratio of the change in the electric charge of a system, to the corresponding change in its electric potential.
- The capacitance of any capacitor can be either fixed or variable depending on their usage. From the equation, it may seem that 'C' depends on charge and voltage. Actually, it depends on the shape and size of the capacitor and also on the insulator used between the conducting plates.

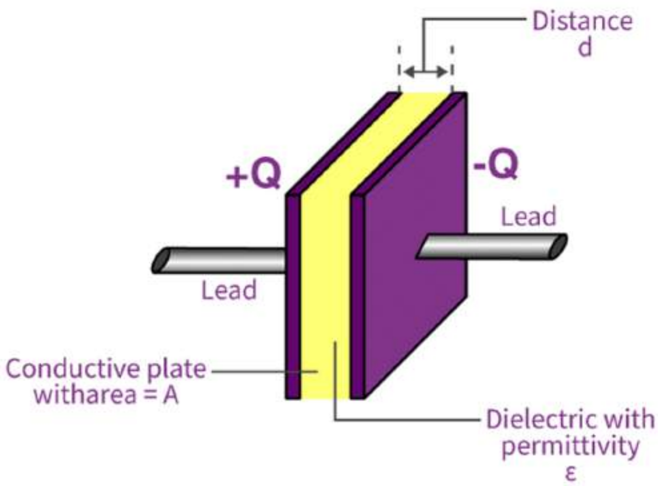
Standard Units of Capacitance

The basic unit of capacitance is Farad. But, Farad is a large unit for practical tasks. Hence, capacitance is usually measured in the sub-units of Farads such as micro-farads (μF) or pico-farads (pF).
Most of the electrical and electronic applications are covered by the following standard unit (SI) prefixes for easy calculations:

- **1 mF (millifarad) = 10^{-3} F**
- **1 μF (microfarad) =10^{-6} F**
- **1 nF (nanofarad) = 10^{-9} F**
- **1 pF (picofarad) = 10^{-12} F**

Different Types of Capacitor

1. Capacitance of a Parallel Plate Capacitor



- The parallel plate capacitor as shown in the figure has two identical conducting plates, each having a surface area A and separated by a distance d. When voltage V is applied to the plates, it stores charge Q.
- The force between charges increases with charge values and decreases with the distance between them. The bigger the area of the plates, the

more charge they can store. Hence, the value of C is greater for a large value of A. Similarly, the closer the plates are, the greater the attraction of the opposite charges on them. There fore C is greater for a smaller d.

- The charge density on the plates is given by the formula:

$$\sigma = \frac{Q}{A}$$

- When the distance of separation (d) is small, the electric field between the plates is fairly uniform and its magnitude is given by:

$$E = \frac{\sigma}{\epsilon_0}$$

- As the electric field between the plates is uniform, the potential difference between the plates is given by

$$v = Ed = \frac{\sigma d}{\epsilon_0} = \frac{Qd}{\epsilon_0 A}$$

- Substituting the above value of V in the capacitance formula, we get

$$C = \frac{Q}{V} = \frac{Q}{Qd/\epsilon_0 A} = \epsilon_0 \frac{A}{d}$$

- The capacitance of a parallel plate capacitor is given by the

formula $$C = \epsilon_0 \frac{A}{d}$$

Example. Calculate the capacitance of an empty parallel-plate capacitor that has metal plates with an area of 1.00 m^2, separated by 1.00 mm?

Solution: Using the formula, we can calculate the capacitance as follows:

$$C = \epsilon_0 \frac{A}{d}$$

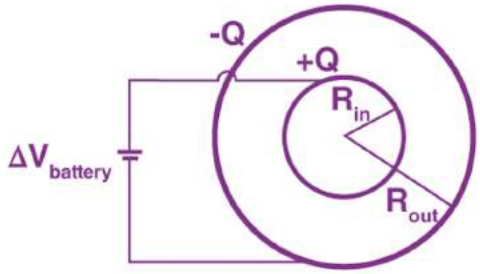
Substituting the values, we get

$$C = (8.85 \times 10^{-12}\ \frac{F}{m})\frac{1\,m^2}{1{\times}10^{-3}\ m} = 8.85 \times 10^{-9}\ F = 8.85\,nF$$

2. Capacitance of a Spherical Capacitor

- Spherical capacitors consist of two concentric conducting spherical shells of radii R_1 and R_2. The shells are given equal and opposite charges +Q and –Q respectively. The electric field between shells is directed radially outward.

- The magnitude of the field can be obtained by applying Gauss law over a spherical Gaussian surface of radius r concentric with the shells.



- The enclosed charge is +Q, therefore

$$\oint_S \vec{E} \cdot \hat{n} dA = E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

- The electric field between the conductor is given as

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

- Integrating $\vec{E}$ along the radial path between the shells, we get

$$V = \int_{R_1}^{R_2} \vec{E} \cdot \overrightarrow{dl} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- The potential difference between two conductors can be calculated using the formula

$$V_B - V_A = -\int_A^B \vec{E}\, \overrightarrow{dL}$$

- The potential difference between the plates is
  $V = -(V_2 - V_1) = V1 - V_2$
- Substituting the value of V in the capacitance formula, we get

$$C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$

- The capacitance of a spherical capacitor is given by the equation

$$C = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$

Example. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is

given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32. Determine the capacitance of the capacitor.

Solution:

Given:

The radius of the inner sphere, $r_2 = 12$ cm $= 0.12$ m

The radius of the outer sphere, $r_1 = 13$ cm $= 0.13$ m

Charge on the inner sphere, $q = 2.5\ \mu C = 2.5 \times 10^{-6}$ C

Dielectric constant of a liquid, $\epsilon_r = 32$

The capacitance of a spherical capacitor is given by the relation:

$$C = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$

ϵ_0 = Permittivity of free space = $8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

Substituting the values in the equation, we get

$$C = \frac{32 \times 0.12 \times 0.13}{9 \times 10^9 (0.13 - 0.12)} \quad C = 5.5 \times 10^{-9} F$$

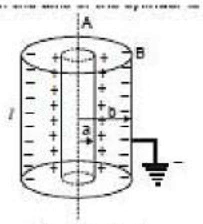
3. Cylindrical Capacitor

- Cylindrical capacitor consists of two co-axial cylinders of radii a and b and length l. If a charge q is given to the inner cylinder, induced change -q will reach the inner surface of the outer cylinder. By symmetry, the electric field in region between the cylinders is radially outwards.
- By Gauss's theorem, the electric field at a distance r from the axis of the cylinder is given by

  $$E = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r}$$

- The potential difference between the cylinders is given by

  $$V = -\int_b^a \vec{E}\,\vec{dr} = -\frac{1}{2\pi\varepsilon_0 l} q \int_b^a \frac{dr}{r} = \frac{-q}{2\pi\varepsilon_0 l}\left(\ln\frac{a}{b}\right)$$



or, $$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\left(\ln\frac{a}{b}\right)}$$

Factors Affecting Capacitance

➤ Dielectric

- The effect of dielectric on capacitance is that the greater the permittivity of the dielectric the greater the capacitance, likewise lesser the permittivity of the dielectric the lesser is the capacitance. Some materials offer less opposition to the field flux for a given amount of field force. Materials with greater permittivity allow more field flux, hence greater charge is collected.

➤ Plate Spacing

- The effect of spacing on the capacitance is that it is inversely proportional to the distance between the plates. Mathematically it is given as:

$$C \propto \frac{1}{d}$$

➤ Area of the Plates

- The effect of the area of the plate is that the capacitance is directly proportional to the area. Larger the plate area more is the capacitance value. Mathematically it is given as:
  $C \propto A$

What are the Applications of Capacitors?

➢ Capacitors for Energy Storage

- Since the late 18th century, capacitors are used to store electrical energy. Individual capacitors do not hold a great deal of energy, providing only enough power for electronic devices to use during temporary power outages or when they need additional power.
- <u>There are many applications that use capacitors as energy sources and a few of them are as follows:</u>
  (i) Audio equipment
  (ii) Camera Flashes
  (iii) Power supplies
  (iv) Magnetic coils
  (v) Lasers

- Supercapacitors are capacitors that have high capacitances up to 2 kF. These capacitors store large amounts of energy and offer new technological possibilities in areas such as electric cars, regenerative braking in the automotive industry and industrial electrical motors, computer memory backup during power loss, and many others.

➢ Capacitors for Power Conditioning

- One of the important applications of capacitors is the conditioning of power supplies. Capacitors allow only AC signals to pass when they are charged blocking DC signals.
- This effect of a capacitor is majorly used in separating or decoupling different parts of electrical circuits to reduce noise, as a result of improving efficiency. Capacitors are also used in utility substations to counteract inductive loading introduced by transmission lines.

➢ Capacitors as Sensors

- Capacitors are used as sensors to measure a variety of things including humidity, mechanical strain, and fuel levels. Two aspects of capacitor construction are used in the sensing application – the distance between the parallel plates and the material between them.
- The former is used to detect mechanical changes such as acceleration and pressure and the latter is used in sensing air humidity.

➢ Capacitors for Signal Processing

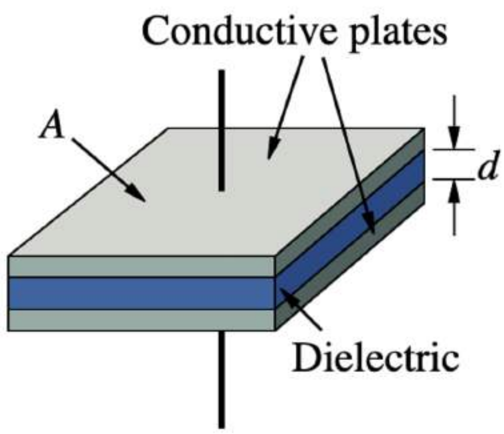
- There are advanced applications of capacitors in information technology. Capacitors are used by Dynamic Random Access Memory (DRAM) devices to represent binary information as bits.
- Capacitors are also used in conjunction with inductors to tune circuits to particular frequencies, an effect exploited by radio receivers, speakers, and analog equalizers.

Parallel Plate Capacitor

What is a Parallel Plate Capacitor?

Parallel Plate Capacitors are formed by an arrangement of electrodes and insulating material or dielectric. A parallel plate capacitor can only store a finite amount of energy before dielectric breakdown occurs. **It can be defined as:**

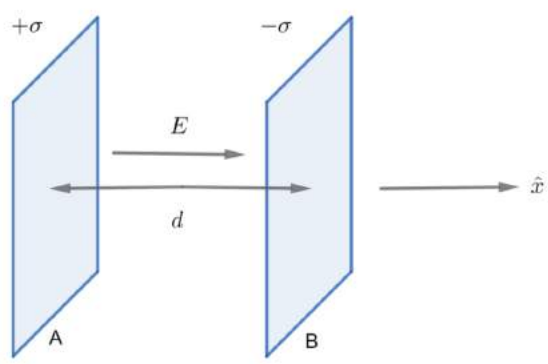
When two parallel plates are connected across a battery, the plates are charged and an electric field is established between them, and this setup is known as the parallel plate capacitor.



Formula

The direction of the electric field is defined as the direction in which the positive test charge would flow. Capacitance is the limitation of the body to store the electric charge. Every capacitor has its capacitance. The typical parallel-plate capacitor consists of two metallic plates of area A, separated by the distance d.
The parallel plate capacitor formula is given by: $C = k\epsilon_0 A/d$
Wher.
ϵ_0 is the permittivity of space (8.854×10^{-12} F/m)
k is the relative permittivity of dielectric material
d is the separation between the plates
A is the area of plates

Derivation



Let the two plates be parallel each other each carrying a surface charge density $+\sigma$ and $-\sigma$ respectively. A is the area of the plates and d is the separation between them.

The electric field of a thin charged plate is given by $E = \frac{\sigma}{2\epsilon_0}$ and is directed normally outwards from the plate. The total electric field between the two plates is given as

$$\vec{E} = \vec{E}_A + \vec{E}_B$$
$$\vec{E} = \frac{+\sigma}{2\epsilon_0}(+\hat{x}) + \frac{-\sigma}{2\epsilon_0}(-\hat{x}) = \frac{\sigma}{\epsilon_0}\hat{x}$$
$$E = |\vec{E}| = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

The potential difference between the two plates is

$$V = E \times d = \frac{Qd}{A\epsilon_0}$$

The capacitance is

$$C = \frac{Q}{V} = \frac{A\epsilon_0}{d}$$

Combination of Capacitors

How Capacitors are connected?

Capacitors combination can be made in many ways. The combination is connected to a battery to apply a potential difference (V) and charge the plates (Q). We can define the equivalent capacitance of the combination between two points to be: $C = Q/V$

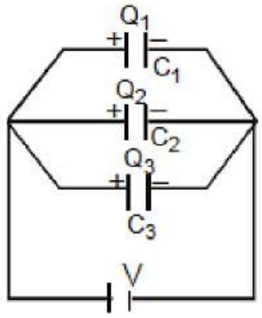
Two frequently used methods of combination are:

- Parallel combination
- Series combination

Parallel Combination of Capacitors

When one plate of one capacitor is connected with one plate of the other capacitor, such combination is called parallel combination.

All capacitors have the same potential difference but different charges.
We can say that: $\mathbf{Q_1 = C_1V}$
Q_1 = Charge on capacitor C_1
C_1 = Capacitance of capacitor C_1



V = Potential across capacitor C_1
The charge on the capacitor is proportional to its capacitance Q µ C

$$Q_1 = \frac{C_1}{C_1 + C_2 + C_3} Q$$

$$Q_2 = \frac{C_2}{C_1 + C_2 + C_3} Q$$

$$Q_3 = \frac{C_3}{C_1 + C_2 + C_3} Q$$

Where $Q = Q_1 + Q_2 + Q_3$ ....
Key Points:

1. The maximum charge will flow through the capacitor of the largest value.
2. Equivalent capacitance of parallel combination, $C_{eq} = C_1 + C_2 + C_3$
3. Equivalent capacitance is always greater than the largest capacitor of combination.
4. Half of the energy supplied by the battery is stored in the form of electrostatic energy and half of the energy is converted into heat through resistance.
5. Energy stored in the combination:

$$V_{combination} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \ldots\ldots\ldots$$

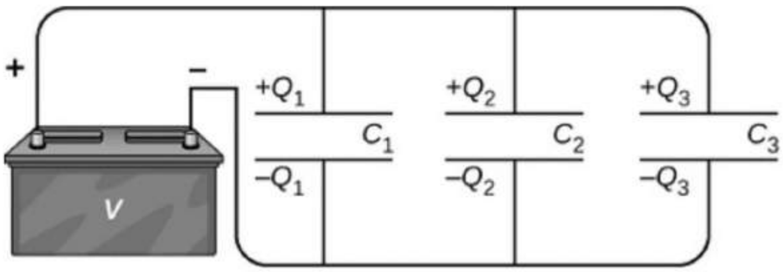
$$= \frac{1}{2}(C_1 + C_2 + C_3\ldots\ldots)V^2 \ \frac{1}{2}C_{eq}V^2$$

$$U_{battery} = QV = CV^2$$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

Formulae Derivation for Parallel combination:

A parallel combination of three capacitors, with one plate of each capacitor connected to one side of the circuit and the other plate connected to the other side, is illustrated in Figure.



Since the capacitors are connected in parallel, they all have the same voltage V across their plates. However, each capacitor in the parallel network may store a different charge. To find the equivalent capacitance Cp of the parallel network, we note that the total charge Q stored by the network is the sum of all the individual charges:

$Q = Q_1 + Q_2 + Q_3$

On the left-hand side of this equation, we use the relation $Q = CpV$, which holds for the entire network. On the right-hand side of the equation, we use the relations $Q_1 = C_1V$, $Q_2 = C_2V$, and $Q_3 = C_3V$ for the three capacitors in the network.

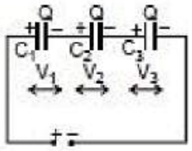
In this way we obtain $CpV = C_1V + C_2V + C_3V$.

This equation, when simplified, is the expression for the equivalent capacitance of the parallel network of three capacitors: $Cp = C_1 + C_2 + C_3$

This expression is easily generalized to any number of capacitors connected in parallel in the network.

Series Combination of Capacitors

When initially uncharged capacitors are connected as shown, then the combination is called series combination



All capacitors will have the same charge but different potential difference across them.

We can say that $V_1 = \frac{Q}{C_1}$

V_1 = potential across C_1
Q = charge on positive plate of C_1
C_1 = capacitance of capacitor similarly
$V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$.........

$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$
We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination.
$V \propto \frac{1}{C}$

Key Points:

1. In a series combination, the smallest capacitor gets maximum potential.
2. $V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots} V$, $V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots} V$, $V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots} V$
   Where $V = V_1 + V_2 + V_3$
3. **Equivalent Capacitance:**
   Equivalent capacitance of any combination is that capacitance which when connected in place of the combination stores the same charge and energy as that of the combination
   In series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ ....
4. In series, the combination equivalent is always less than the smallest capacitor of the combination.
5. Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance.

6. **Energy stored in the combination:**

$U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$

$U_{combination} = \frac{Q^2}{2C_{eq}}$

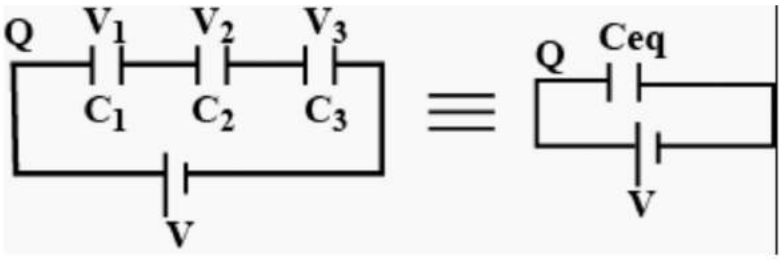
The energy supplied by the battery in charging the combination

$U_{battery} = Q \times V = Q . \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$

$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$

Formulae Derivation for Series combination:

Let the capacitance of each capacitor be C_1, C_2 and C_3 and their equivalent capacitance is C_{eq}.
As these capacitors are connected in series, thus charge across each capacitor is same as Q. When some electrical components, let say 3, are connected in series with each other, the potential difference of the battery V gets divided across each component as
V_1, V_2 and V_3 as shown in the figure.



∴ $V = V_1 + V_2 + V_3$
Using $V = Q/C$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Equivalent capacitance for series combination

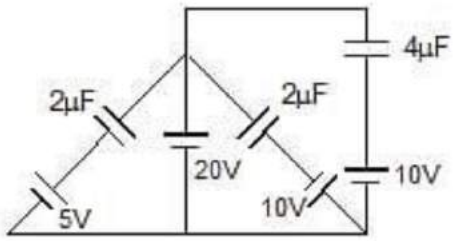
$$= \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

In general,

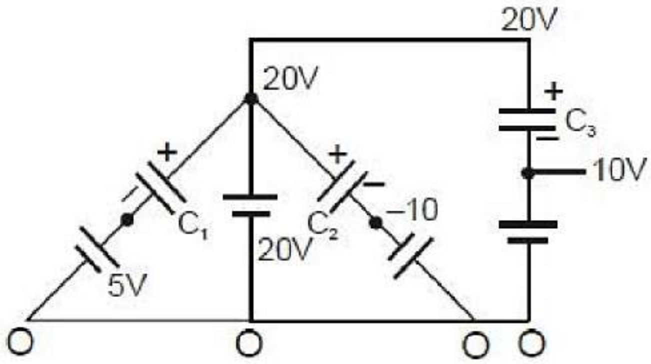
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots\ldots \frac{1}{C_n}$$

Solved Examples:

Example 1: Find charge on each capacitor.



Sol. Charge on $C_1 = C_1V_1 = 2 \times (20 - 5)\mu C$



$= 30\ \mu C$

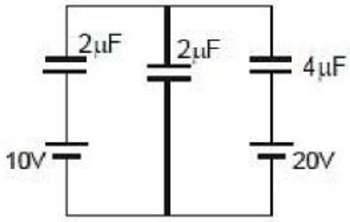
Charge on $C_2 = C_2V_2 = 2 \times (20 - (-10))\ \mu C$

$= 60\ \mu C$

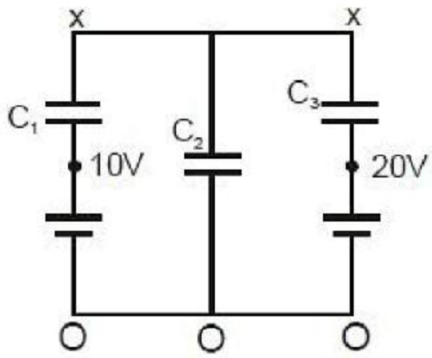
Charge on $C_3 = C_3V_3 = 4 \times (20 - 10)\ \mu C$

$= 40\ \mu C$

Example 2: Find charge on each capacitor.

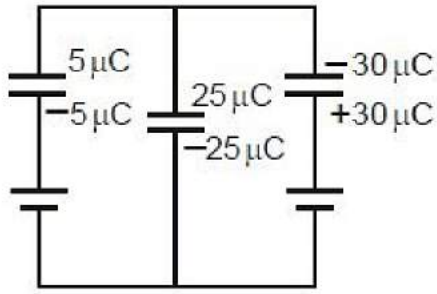


Sol. Charge on $C_1 = (x - 10)\ C_1$
Charge on $C_2 = (x - 0)\ C_2$
Charge on $C_3 = (x - 20)\ C_3$
Now from charge conservation at node x



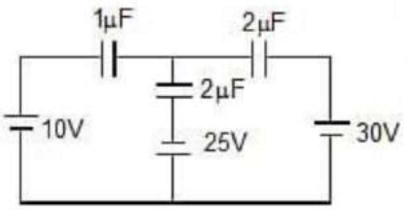
$(x - 10)\ C_1\ (x - 0)\ C_2\ (x - 20)\ C_3 = 0$
$\Rightarrow 2x - 20\ 2x\ 4x - 80 = 0$
$\Rightarrow x = 25$ Therefore
so
$Q_{C_1} = \left(\frac{25}{2} - 10\right) 2\mu C = 5\mu C$

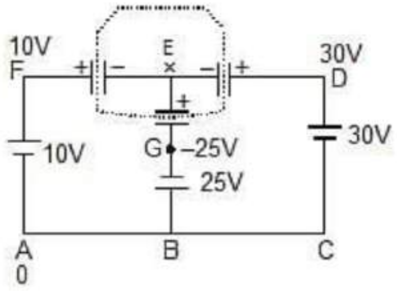
$Q_{C_2} = \frac{25}{2} \times 2\mu C = 25\,\mu C$



$Q_{C_3} = (25/2 - 40) \times 4\mu C = -30\mu C$

Example 3: In the given circuit find out the charge on each capacitor. (Initially they are uncharged)





Sol. Let potential at A is 0, so at D it is 30 V, at F it is 10 V and at point G potential is -25V. Now apply Kirchhoff's I[st] law at point E. (total charge of all the plates connected to 'E' must be same as before i.e. 0)

Therefore, (x - 10) + (x - 30) 2 +(x 25) 2 = 0

5x = 20

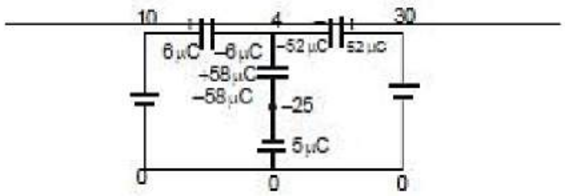
x = 4

Final charges:

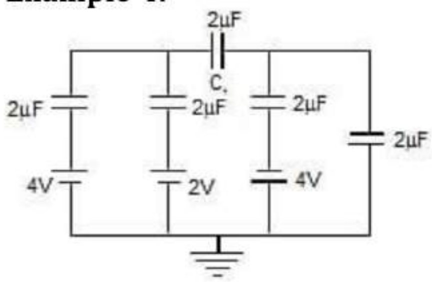
Q_{2mF} = (30 - 4) 2 = 52 mC

Q_{1mF} = (10 - 4) = 6 mC

Q_{2mF} = (4 - (-25)) 2 = 58 mC

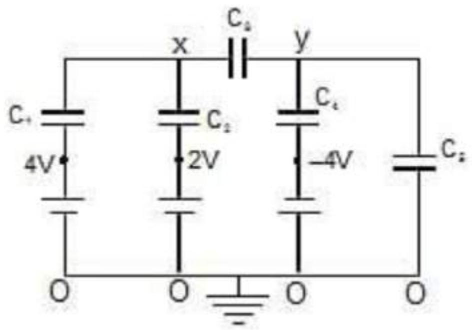


Example 4:



Find voltage across capacitor C_1.

Sol.



Now from charge conservation at node x and y

for x

$(x - 4)\, C_1 + (x - 2)\, C_2 + (x - y)\, C_3 = 0 \Rightarrow$

$2(x - 4) + 2(x - 2)\ (x - y)\ 2 = 0$

$6x - 2y - 12 = 0$ .....(1)

For y

$(y - x)\, C_3 + [y - (-4)]C_4\ (y - 0)\, C_5 = 0 \Rightarrow (y - x)2\ (y\ 4)\ 2\ y\ 2 = 0$

$= 6y - 2x\ 8 = 0$..... (2)

eq. (1) & (2)

$y = -3$ Therefore

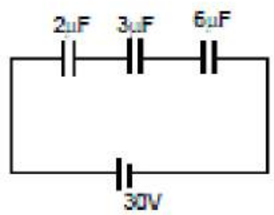
$x = 7$ Therefore

So potential difference $= x - y = \frac{7}{4} - \left(-\frac{3}{4}\right) = \frac{5}{3}$ volt

Example 5: Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30V. Find out following:

(i) charge flow through the battery,

(ii) potential energy in 3 mF capacitor.



(iii) U_{total} in capacitors

(iv) heat produced in the circuit

Sol. $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$

$C_{eq} = 1\ \mu F.$

(i) $Q = C_{eq} V = 30\ \mu C$

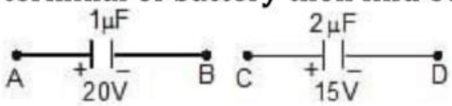
(ii) charge on 3μF capacitor = 30 μC

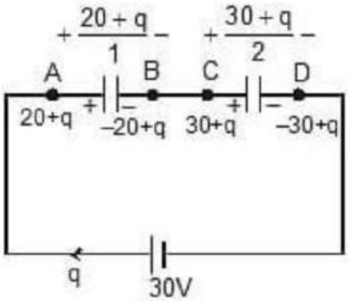
energy = $\frac{Q^2}{2C} = \frac{30 \times 30}{2 \times 3} = 150$ μJ

(iii) $U_{total} = \frac{30 \times 30}{2}$ μJ $= 450$ μJ

(iv) Heat produced = (30 μC) (30) - 450 μJ = 450 μJ

Example 6: Two capacitors of capacitance 1 mF and 2mF are charged to potential difference 20 V and 15 V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor.





Now applying kirchhoff voltage law

$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$

- 40 - 2q - 30 - q = - 60

3q = - 10

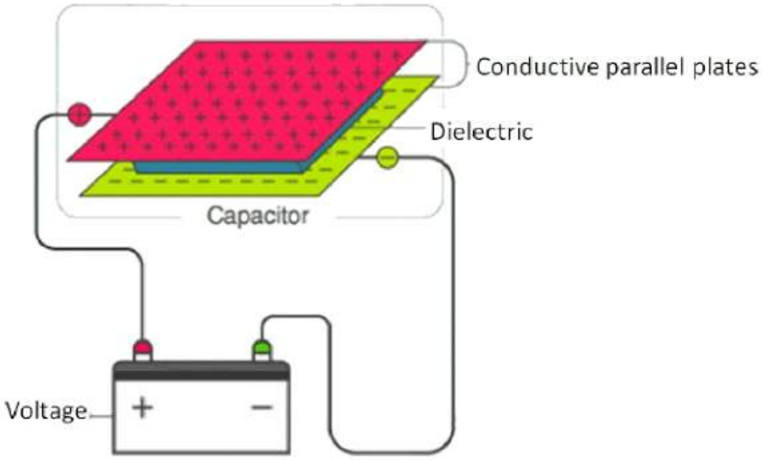
Charge flow = - $\frac{10}{3}$ μC.

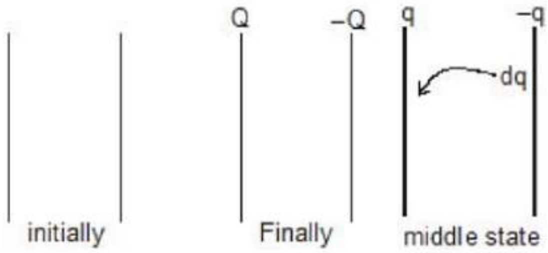
Charge on capacitor of capacitance 1μF = 20 q = $\frac{50}{3}$

Charge on capacitor of capacitance 2μF = 30 q = $\frac{80}{3}$

Energy Stored in a Capacitor

Energy Stored in a Charged capacitor





Work has to be done in charging a conductor against the force of repulsion by the already existing charges on it. The work is stored as a potential energy in the electric field of the conductor. Suppose a conductor of capacity C is charged to a potential V_0 and let q_0 be the charge on the conductor at this instant. The potential of the conductor when (during charging) the charge on it was q ($< q_0$) is,

$$V = \frac{q}{C}$$

Now, work done in bringing a small charge dq at this potential is,

$$dW = Vdq = \left(\frac{q}{C}\right)dq$$

Therefore, total work done in charging it from 0 to q_0 is,

$$W = \int_0^{q_0} dW = \int_0^{q_0} \frac{q}{C} dq = \frac{1}{2}\frac{q_0^2}{C}$$

This work is stored as the potential energy,

Therefore, $$U = \frac{1}{2}\frac{q_0^2}{C}$$

Further by using $q_0 = CV_0$ we can write this expression also as,

$$U = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

In general if a conductor of capacity C is charged to a potential V by giving it a charge q, then

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

3.1 Energy Density of a Charged Capacitor

This energy is localized on the charges or the plates but is distributed in the field. Since in case of a parallel plate capacitor, the electric field is only between the plates, i.e., in a volume (A × d), the energy density

$$U_E = \frac{U}{\text{volume}} = \frac{\frac{1}{2}CV^2}{A \times d} = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\frac{V^2}{Ad}$$

or $$U_E = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2 \quad \left[\because \frac{V}{d} = E\right]$$

3.2 Calculation of Capacitance

The method for the calculation of capacitance involves integration of the electric field between two conductors or the plates which are just equipotential surfaces to obtain the potential difference V_{ab}. Thus,

$$V_{ab} = -\int_b^a \vec{E}.\vec{dr}$$

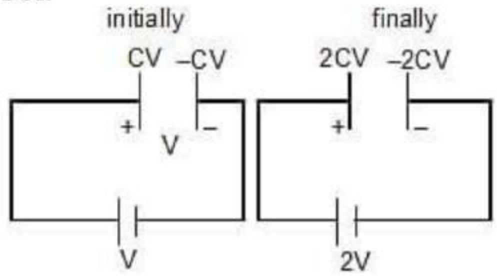
Therefore, $$C = \frac{q}{V_{ab}} = \frac{q}{-\int_b^a \vec{E}.\vec{dr}}$$

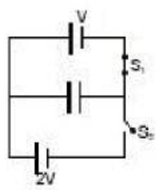
3.3 Heat Generated:

(1) Work done by battery
$W = QV$
Q = charge flow in the battery
V = EMF of battery
(2) $W = Ve$ (When Battery discharging)
$W = -Ve$ (When Battery charging)
(3) $\because$ $Q = CV$ (C = equivalent capacitance)
so $W = CV \times V = CV^2$

Now energy on the capacitor $= \frac{1}{2}CV^2$
Therefore, Energy dissipated in form of heat (due to resistance)
H = Work done by battery - {final energy of capacitor - initial energy of capacitor}

Ex.3 At any time S_1 switch is opened and S_2 is closed then find out heat generated in circuit.
Sol.





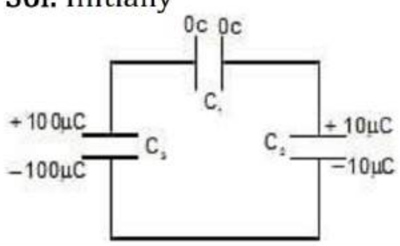
Charge flow through battery $= Q_f - Q_i$
$= 2CV - CV = CV$

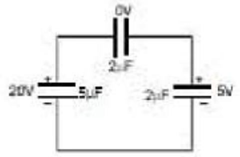
$$H = (CV \times 2V) - \left\{\frac{1}{2}C(2V)^2 - \frac{1}{2}CV^2\right\} = 2CV^2 - \left\{2CV^2 - \frac{1}{2}CV^2\right\}$$

$$H = \frac{1}{2}CV^2$$

Ex.4 (a) Find the final charge on each capacitor if they are connected as shown in the figure.

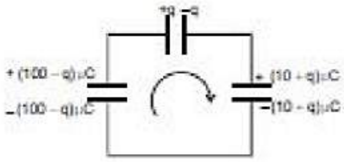
Sol. Initially





Finally let q charge flows clockwise then
Now applying KVL

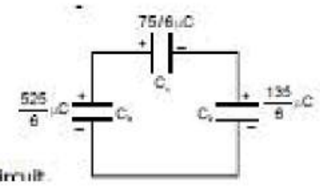
$$\frac{+q}{C_1}+\frac{(10+q)}{C_2}-\frac{(100-q)}{C_3}=0$$



$$\Rightarrow \quad \frac{q}{2}+\frac{10+q}{2}-\frac{100-q}{5}=0$$

5q + 50 + 5q - 200 + 2q = 0

12 q - 150 = 0 ⇒ $q=\frac{75}{6}\mu C$
so finally

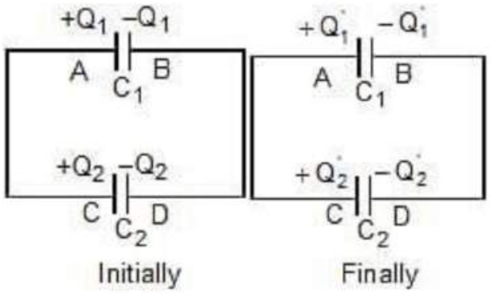


(b) Find heat loss in the above circuit.
ΔH = Energy [initially - finally] on capacitor

$$=\left[\left\{\frac{1}{2}\times5\times(20)^2+\frac{1}{2}\times2\times(5)^2\right\}-\left\{\frac{1}{2}\times\left(\frac{525}{6}\right)^2\times\frac{1}{5}+\frac{1}{2}\times\left(\frac{75}{6}\right)^2\times\frac{1}{2}+\frac{1}{2}\left(\frac{135}{6}\right)^2\times\frac{1}{2}\right\}\right]\times10^{-6}J$$

Distribution of Charges on Connecting two Charged Capacitors:

When two capacitors C_1 and C_2 are connected as shown in figure



Before connecting the capacitors		
Parameter	**Ist Capacitor**	**IInd Capacitor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the capacitors		
Parameter	**Ist Capacitor**	**IInd Capacitor**
Capacitance	C_1	C_2
Charge	Q_1'	Q_2'
Potential	V_1	V_2

(a) Common potential:
By charge conservation on plates A and C before and after connection.
$Q_1 + Q_2 = C_1V + C_2V$

$$\Rightarrow V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $$Q_1' = C_1V = \frac{C_1}{C_1+C_2}(Q_1+Q_2)$$

$$Q_2' = C_2V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

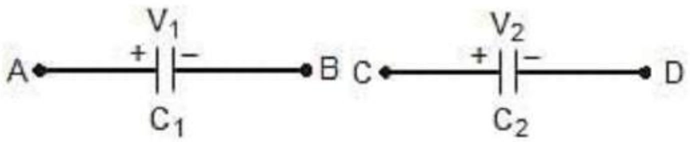
(c) Heat loss during redistribution:

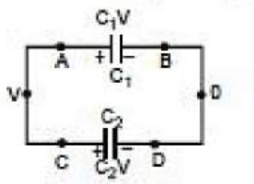
$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

- When plates of similar charges are connected with each other (with and - with -) then put all values (Q_1, Q_2, V_1, V_2) with positive sign.
- When plates of opposite polarity are connected with each other (with -) then take charge and potential of one of the plate to be negative.

Derivation of above formulae:





Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

$C_1V + C_2V = C_1V_1 + C_2V_2$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

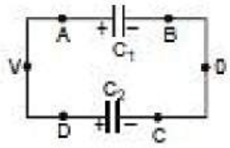
$$H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right]$$

$$= \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$



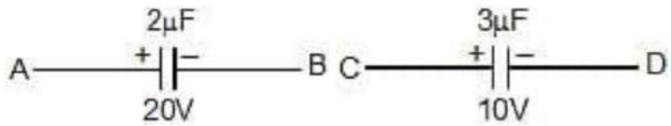
when oppositely charged terminals are connected then

Therefore, $C_1V\ C_2V = C_1V_1 - C_2V_2$

$$V = \frac{C_1V_1 - C_2V_2}{C_1+C_2}$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 + V_2)^2$$

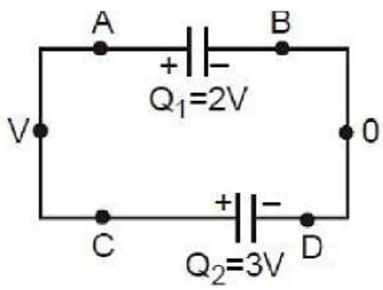
Ex.5 Find out the following if A is connected with C and B is connected with D.
(i) How much charge flows in the circuit.
(ii) How much heat is produced in the circuit.



Sol. Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

By charge conservation,

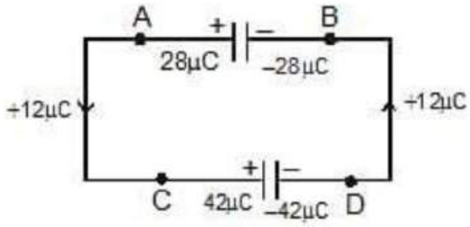
$3V + 2V = 40 + 30$



$5V = 70 \Rightarrow V = 14$ volt

Charge flow = 40 - 28 = 12 μC

Now final charges on each plate is shown in the figure.



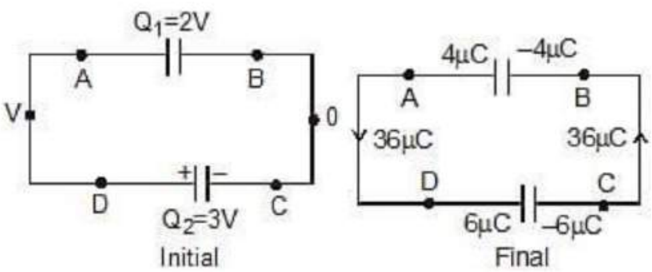
(ii) Heat produced $= \frac{1}{2} \times 2 \times (20)^2 \;\; \frac{1}{2} \times 3 \times (10)^2 - \frac{1}{2} \times 5 \times (14)^2$

$= 400 \; 150 - 490$

$= 550 - 490 = 60$ mJ

- When capacitor plates are joined then the charge remains conserved.
- We can also use direct formula of redistribution as given above.

Ex.6 Repeat above question if A is connected with D and B is connected with C.



Sol. Let potential of B and C is zero and common potential on capacitors is V, then at A and D it will be V

$2V + \;\; 3V = 10 \Rightarrow V = 2$ volt

Now charge on each plate is shown in the figure.

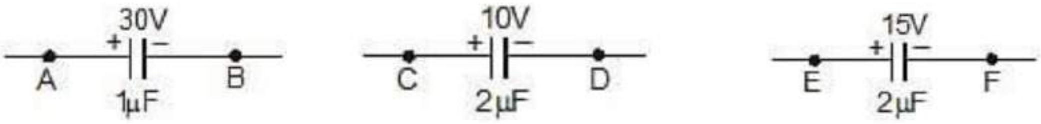
Heat produced $= 400 + 150 - 1$

Therefore $2 \times 5 \times 4$

$= 550 - 10 = 540\ \mu J$

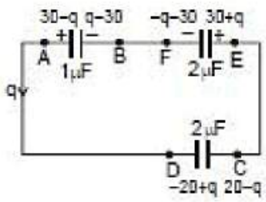
- Here heat produced is more. Think why?

Ex.7 Three capacitors as shown of capacitance 1mF, 2mF and 2mF are charged upto potential difference 30 V, 10 V and 15V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.

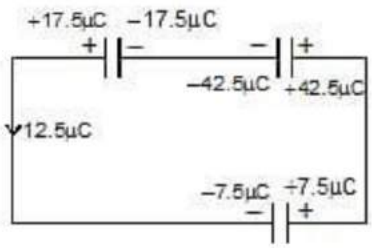


Sol. Let charge flow is q.
Now applying Kirchhoff's voltage low

$$-\frac{(q-20)}{2}-\frac{(30+q)}{2}+\frac{30-q}{1}=0$$



- 2q = - 25
q = 12.5 mC
Final charges on plates



Van De Graaff Generator

Theory:
The American physicist, Dr. Robert Jemison Van de Graaff invented the Van de Graaff generator in 1931. The device has the ability to produce extremely high voltages - as high as 20 million volts. Van de Graaff invented the generator to supply the high energy needed for early particle accelerators. These accelerators are known as atom smashers because they accelerates the sub atomic particles to very high speeds and then "smash" them in to the target atoms. The resulting collision creates other sub atomic particles and high energy radiations such as X-rays. The ability to create these high energy collisions is the foundation of particle and nuclear physics.

Working of the generator is based on two principles:

1. Discharging action of sharp points, ie., electric discharge takes place in air or gases readily, at pointed conductors.
2. If the charged conductor is brought in to internal contact with a hollow conductor, all of its charge transfers to the surface of the hollow conductor no matter how high the potential of the latter may be.

Theory behind construction:

If we have a large conducting spherical shell of radius 'R' on which we place a charge Q, it spreads itself uniformly all over the sphere. The field outside the sphere is just that of a point charge Q at the centre, while the field inside the sphere vanishes. So the potential outside is that of point charge and inside it is constant.
The potential inside the conducting sphere =

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Now suppose that we introduce a small sphere of radius 'r', carrying a charge q, into the large one and place it at the centre. The potential due to this new charge has following values.
Potential due to small sphere of radius r carrying charge

$$q = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Potential at the surface of large shell of radius R

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Taking both charges q and Q in to account we have for the total potential V and the potential difference given by,

$$V(R) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

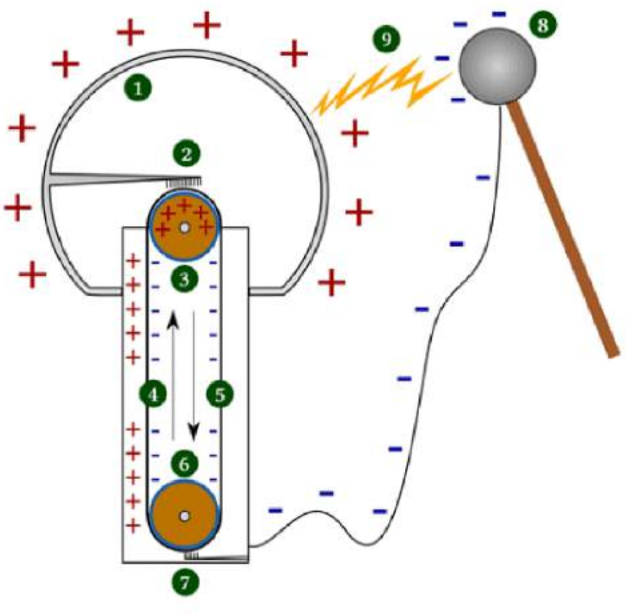
$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

$$V(r) - V(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Now assume that q is positive. We see that, independent of the amount of charge Q that may have accumulated on the larger sphere, it is always at a higher potential: the difference V(r) - V(R) is positive. The potential due to Q is constant upto radius R and so cancels out in the difference.

This means that if we connect the smaller and larger sphere by a wire, the charge q on the former will immediately flow on to the matter, even though the charge Q may be quite large. The natural tendency is for positive charge to move higher to lower potential. Thus, provided we are somehow able to introduce the small charged sphere into the larger one, we can in this way pile up larger and larger amount of charge on the latter. The potential of the outer sphere would also keep rising, at least until it reaches the breakdown field of air.

Construction:



1. hollow metal sphere
2. upper electrode
3. upper roller (for example an acrylic glass)
4. side of the belt with positive charges
5. opposite side of belt, with negative charges
6. lower roller (metal)
7. lower electrode (ground)
8. spherical device with negative charges
9. spark produced by the difference of potentials

It consists of a large metal sphere mounted on high insulating supports. An endless belt b, made of insulating material such as rubber, passes over the vertical pulleys P1 and P2. The pulley P2 is at the centre of the metal sphere and the pulley P1 is vertically below P2. The belt is run by an electric motor M. B1and B2 are two metal brushes called collecting combs.

The positive terminal of a high tension source (HT) is connected to the comb B1. Due to the process called action of points, charges are accumulated at the pointed ends of the comb, the field increases and ionizes the air near them. The positive charges in air are repelled and get deposited on the belt due to corona discharge. The charges are carried by the belt upwards as it moves. When the positively charged portion of the belt comes in front of the brush B2, by the same process of action of points and corona discharge occurs and the metal sphere acquires positive charges. The positive charges are uniformly distributed over the surface of the sphere. Due to the action of points by the negative charges carried by the gas in front of the comb B2, the positive charge of the belt is neutralized. The uncharged portion of the belt returns down collects the positive charge from B1 which in turn is collected by B2. The charge transfer process is repeated. As more and more positive charges are imparted to the sphere, its positive potential goes on rising until a surface maximum is reached. If the potential goes beyond this, insulation property of air breaks down and the sphere gets discharged. The breakdown of air takes place in an enclosed steel chamber filled with nitrogen at high pressure.

Solved Example

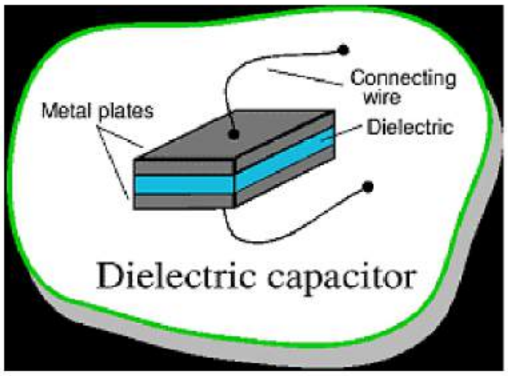
Q: An efficient Van de Graff generator has a rounded terminal as it:
A. Maximises the electric field around it.
B. Minimizes the electric field around it.
C. Disrupts the electric field.
D. Shields it from the electric field.

Solution: A) The whole purpose of the generator is to retain charge and create a region of a high Potential. To prevent leakage of charge through corona discharge or other such phenomena, we have to avoid using spikey or narrow conductors. As a result, the terminals are rounded.

Dielectrics & Polarisation

What is a Dielectric?

Dielectrics are non-conducting substances. They are the insulating materials and are bad conductors of electric current. Dielectric materials can hold an electrostatic charge while dissipating minimal energy in the form of heat. Examples of dielectric are Mica, Plastics, Glass, Porcelain and Various Metal Oxides. You must also remember that even dry air is also an example of a dielectric.



Classification of Dielectrics

Dielectrics are of two types

- **Polar Molecules:** Polar Molecules are those type of dielectric where the possibilities of the positive and negative molecules coinciding with each other are null or zero. This is because they all are asymmetric in shape. Examples: H_2O, CO_2, NO_2 etc.
  When the electric field is not present, it causes the electric dipole moment of these molecules in a random direction. This is why the average dipole moment is zero. If the external electric field is present, the molecules assemble in the same direction as the electric field.
- **Non-Polar Molecule:** Unlike polar molecules, in non-polar molecules, the centre of positive charge and negative coincide, that is it is not zero. The molecule then has no permanent (or intrinsic) dipole moment. Examples: O_2, N_2, H_2 etc.

Induced Electric Dipole Moment

When we apply an external electric field in a non-polar molecule, all the protons travel towards the direction of the electric field and electrons in opposite direction. Due to the presence of an electric field, this process continues unless the internal forces balance them.

Due to this, there is a creation of two centres of charge. They are Polarised and we call them as the Induced Electric Dipole. The dipole moment is the Induced Electric Dipole Moment.

Polarizability

Applied field is directly proportional to induced dipole moment and is independent of the temperature. The direction of induced dipole moment (x) is parallel to the

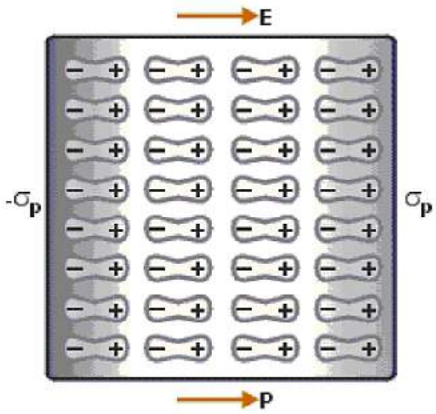
direction of electric field Ê and for a single polar atom. The Polarisability determines the dynamical response of a bound system to external fields.
It also provides an insight into a molecule's internal structure. In a solid, polarizability is the dipole moment per unit volume of the crystal cell:

$$\vec{P} = \epsilon_0 \alpha \vec{E}$$

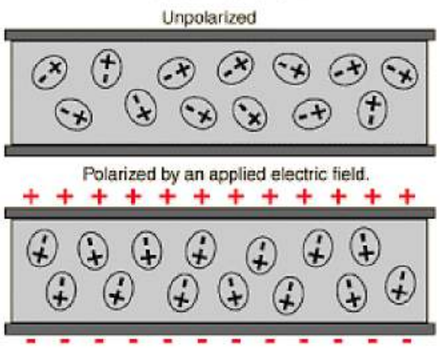
where 'a' is the Atomic Polarisability. The S.I. unit of polarizability is m3 and its dimensions are the same as it's volume.

Electric Polarization

When we place a dielectric slab is in an electric field, then the molecule gains the dipole moment. In such cases, we say that the dielectric is polarised. The Electric Polarization is dipole moment per unit volume of a dielectric material. The polarization is denoted by P.



Polarisation



Polarisation Process

Dielectric Constant

When we place a dielectric slab between the parallel plates, the ratio of the applied electric field strength to the strength of the reduced value of electric field capacitor is the Dielectric Constant. The formula is:

$$k = \frac{E_0}{E}$$

E is always less than or equal to E where E_o is dielectric and E is the net field. The larger the dielectric constant, the more charge can be stored. Completely filling the space between capacitor plates with a dielectric increase the capacitance by a factor of the dielectric constant. $C = {}_{\kappa}C_o$, where C_o is the capacitance with no dielectric between the plates.

Dielectric Strength

For an insulating material, the dielectric strength is the maximum electric field strength that it can withstand intrinsically without experiencing failure of its insulating properties.

Material	Dielectric constant	Dielectric Strength (kV/mm)
Vacuum	1.00000	-
Air (dry)	1.00059	3
Polystyrene	2.6	24
Paper	3.6	16
Water	80	-

Dielectric Polarization

When we apply an external electric field to a dielectric material, we get the Dielectric Polarization. It is the displacement of charges (positive and negative) upon applying an electric field. The main task of the dielectric polarization is to relate macroscopic properties to microscopic properties.
Polarization occurs through the action of an electric field or other external factors, such as mechanical stress, as in the case of piezoelectric crystals. Piezoelectric crystals are those solid materials which accumulate electric charge within them. Dielectric Polarization can also arise spontaneously in pyroelectric crystals, particularly in ferroelectrics. Ferroelectricity is a property of certain materials that have a spontaneous electric polarization that can be reversed by the application of an external electric field.

Solved Example for You
Question: Which of the following is/are non-polar dielectrics?

(a) HCl
(b) Water
(c) Benzene
(d) NH_3

Solution: Option C – Benzene. Ammonia and HCl are polar molecules since they have a net dipole moment towards a particular direction. Both water and benzene are non-polar molecules. But water is a conductor of electricity, whereas benzene is a dielectric (insulator).